Arithmetic Progressions

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 5.1

Q. 1. *In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?*

(i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

(iv) The amount of money in the account every year, when ₹ 10,000 is deposited at compound interest at 8% per annum.

Sol. (*i*) Let us consider,

The first term (T_1) = Fare for the first km = ₹ 15 since, the taxi fare beyond the first km is ₹ 8 for each additional km. $\Rightarrow T_1 = 15$

$\therefore$ Fare for 2 km = ₹ 15 + 1 × ₹ 8 $\Rightarrow T_2 = a + 8$ [*where a = 15*]

Fare for 3 km = ₹ 15 + 2 × ₹ 8 $\Rightarrow T_3 = a + 16$

Fare for 4 km = ₹ 15 + 3 × ₹ 8 $\Rightarrow T_4 = a + 24$

Fare for 5 km = ₹ 15 + 4 × ₹ 8 $\Rightarrow T_5 = a + 32$

⋮

Fare for n km = ₹ 15 + (n – 1) 8 $\Rightarrow T_n = a + (n - 1)\, 8$

We see that above terms **form an A.P.**

(*ii*) Let the amount of air in the cylinder = x

$\therefore$ Air removed in 1st stroke = $\frac{1}{4}x$

$\Rightarrow$ Air left after 1st stroke $= x - \frac{1}{4}x = \frac{3x}{4}$

Air left after 2nd stroke $= \frac{3x}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9}{16}x$

Air left after 3rd stroke $= \frac{9}{16}x - \frac{1}{4}\left(\frac{9}{16}x\right) = \frac{9x}{16} - \frac{9x}{64} = \frac{27x}{64}$

Air left after 4th stroke $= \frac{27}{64}x - \frac{1}{4}\left(\frac{27x}{64}\right) = \frac{27x}{64} - \frac{27}{256} = \frac{91x}{256}$

Thus, the terms are:

$x, \ \frac{3x}{4}, \ \frac{9}{16}x, \ \frac{27}{64}x, \ \frac{91x}{256}$

Here, $\frac{3x}{4} - x = \frac{-x}{4}$

$\frac{9}{16}x - \frac{3x}{4} = \frac{-3x}{16}$

Since $\left(\frac{-x}{4}\right) \neq \left(\frac{-3x}{16}\right)$

The above terms are **not in A.P.**

(*iii*) Here, The cost of digging for 1st metre = ₹ 150

The cost of digging for first 2 metres = ₹ 150 + ₹ 50 = ₹ 200

The cost of digging for first 3 metres = ₹ 150 + (₹ 50) × 2 = ₹ 250

The cost of digging for first 4 metres = ₹ 150 + (₹ 50) × 3 = ₹ 300

$\therefore$ The terms are: 150, 200, 250, 300, ...

Since, 200 – 150 = 50

And 250 – 200 = 50

$\Rightarrow$ (200 – 150) = (250 – 200)

$\therefore$ The above terms **form an A.P.**

(*iv*) $\because$ The amount at the end of 1st year $= 10000\left(1 + \frac{8}{100}\right)^1$

The amount at the end of 2nd year $= 10000\left(1 + \frac{8}{100}\right)^2$

The amount at the end of 3rd year $= 10000\left(1 + \frac{8}{100}\right)^3$

The amount at the end of 4th year $= 10000\left(1 + \frac{8}{100}\right)^4$

$\vdots$

$\therefore$ The terms are

$[10000], \ \left[10000\left(1 + \frac{8}{100}\right)\right], \ \left[10000\left(1 + \frac{8}{100}\right)^2\right], \ \left[10000\left(1 + \frac{8}{100}\right)^3\right], ...$

Obviously,

$$\left[10000\left(1 + \frac{8}{100}\right)\right] - [10000] \neq \left[10000\left(1 + \frac{8}{100}\right)^2\right] - \left[10000\left(1 + \frac{8}{100}\right)\right]$$

$\therefore$ The above terms are **not in A.P.**

Q. 2. *Write first four terms of the AP, when the first term a and the common difference d are given as follows:*

(i) $a = 10, \quad d = 10$ *(ii)* $a = -2, \quad d = 0$

(iii) $a = 4, \quad d = -3$ *(iv)* $a = -1, \quad d = \frac{1}{2}$

(v) $a = -1.25, \quad d = -0.25$

Sol. (*i*) $\because T_n = a + (n-1)d$

$\therefore$ For $a = 10$ and $d = 10$, we have:

$$T_1 = 10 + (1-1) \times 10 = 10 + 0 = 10$$
$$T_2 = 10 + (2-1) \times 10 = 10 + 10 = 20$$
$$T_3 = 10 + (3-1) \times 10 = 10 + 20 = 30$$
$$T_4 = 10 + (4-1) \times 10 = 10 + 30 = 40$$

Thus, the first four terms of A.P. are:

10, 20, 30, 40.

(*ii*) $\because T_n = a + (n-1)d$

$\therefore$ For $a = -2$ and $d = 0$, we have:

$$T_1 = -2 + (1-1) \times 0 = -2 + 0 = -2$$
$$T_2 = -2 + (2-1) \times 0 = -2 + 0 = -2$$
$$T_3 = -2 + (3-1) \times 0 = -2 + 0 = -2$$
$$T_4 = -2 + (4-1) \times 0 = -2 + 0 = -2$$

$\therefore$ The first four terms are:

– 2, – 2, – 2, – 2.

(*iii*) $\because T_n = a + (n-1)d$

$\therefore$ For $a = 4$ and $d = -3$, we have:

$$T_1 = 4 + (1-1) \times (-3) = 4 + 0 = 4$$
$$T_2 = 4 + (2-1) \times (-3) = 4 + (-3) = 1$$
$$T_3 = 4 + (3-1) \times (-3) = 4 + (-6) = -2$$
$$T_4 = 4 + (4-1) \times (-3) = 4 + (-9) = -5$$

Thus, the first four terms are:

4, 1, – 2, – 5.

(*iv*) $\because T_n = a + (n-1)d$

For $a = -1$ and $d = \frac{1}{2}$, we get

$$T_1 = -1 + (1-1) \times \frac{1}{2} = -1 + 0 = -1$$
$$T_2 = -1 + (2-1) \times \frac{1}{2} = -1 + \frac{1}{2} = -\frac{1}{2}$$
$$T_3 = -1 + (3-1) \times \frac{1}{2} = -1 + 1 = 0$$
$$T_4 = -1 + (4-1) \times \frac{1}{2} = -1 + \frac{3}{2} = \frac{1}{2}$$

$\therefore$ The first four terms are:

$$\mathbf{-1, \quad -\frac{1}{2}, \quad 0, \quad \frac{1}{2}.}$$

(*v*) $\because$ $T_n = a + (n-1)d$

$\therefore$ For $a = -1.25$ and $d = -0.25$, we get

$T_1 = -1.25 + (1-1) \times (-0.25) = -1.25 + 0 = -1.25$

$T_2 = -1.25 + (2-1) \times (-0.25) = -1.25 + (-0.25) = -1.50$

$T_3 = -1.25 + (3-1) \times (-0.25) = -1.25 + (-0.50) = -1.75$

$T_4 = -1.25 + (4-1) \times (-0.25) = -1.25 + (-0.75) = -2.0$

Thus, the four terms are:

– 1.25, – 1.50, – 1.75, – 2.0

Q. 3. *For the following APs, write the first term and the common difference:*

(*i*) 3, 1, – 1, – 3, ... (*ii*) – 5, – 1, 3, 7, ...

(*iii*) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$ (*iv*) 0.6, 1.7, 2.8, 3.9, ...

Sol. (*i*) We have: 3, 1, – 1, – 3, ...

$\Rightarrow$ $T_1 = 3 \Rightarrow a = 3$

$T_2 = 1$

$T_3 = -1$

$T_4 = -3$

$\therefore$ $\left.\begin{array}{l} T_2 - T_1 = 1 - 3 = -2 \\ T_4 - T_3 = -3 - (-1) = -3 + 2 = -2 \end{array}\right\} \Rightarrow d = -2$

Thus, $\boldsymbol{a = 3}$ and $\boldsymbol{d = -2}$

(*ii*) We have: – 5, – 1, 3, 7, ...

$\Rightarrow$ $T_1 = -5 \Rightarrow a = -5$

$T_2 = -1 \Rightarrow d = T_2 - T_1 = -1 - (-5) = -1 + 5 = 4$

$T_3 = 3 \Rightarrow d = -1 + 5 = 4$

$T_4 = 7$ $\quad T_4 - T_3 = 7 - 3 = 4 \} \Rightarrow d = 4$

Thus, $\boldsymbol{a = -5}$ and $\boldsymbol{d = 4}$

(*iii*) We have: $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

$\Rightarrow$ $T_1 = \frac{1}{3} \Rightarrow a = \frac{1}{3}$

$T_2 = \frac{5}{3} \Rightarrow d = T_2 - T_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

$\left.\begin{array}{l} T_3 = \frac{9}{13} \\ T_4 = \frac{13}{3} \end{array}\right\} \Rightarrow d = T_4 - T_3 = \frac{13}{3} - \frac{9}{3} = \frac{4}{3}$

Thus, $\boldsymbol{a = \frac{1}{3}}$ and $\boldsymbol{d = \frac{4}{3}}$

(*iv*) We have: 0.6, 1.7, 2.8, 3.9, ...

$\Rightarrow$ $T_1 = 0.6 \Rightarrow a = 0.6$

$T_2 = 1.7 \Rightarrow d = T_2 - T_1 = 1.7 - 0.6 = 1.1$

$T_3 = 2.8$

$T_4 = 3.9 \Rightarrow d = T_4 - T_3 = 3.9 - 2.8 = 1.1$

Thus, $\boldsymbol{a = 0.6}$ and $\boldsymbol{d = 1.1}$

Q. 4. *Which of the following are APs? If they form an AP, find the common difference d and write three more terms.*

(i) 2, 4, 8, 16, ... (ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

(iii) – 1.2, – 3.2, – 5.2, – 7.2, ... (iv) – 10, – 6, – 2, 2, ...

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$ (vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) 0, – 4, – 8, – 12, ... (viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$

(ix) 1, 3, 9, 27, ... (x) a, 2a, 3a, 4a, ...

(xi) $a, a^2, a^3, a^4, \ldots$ (xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$ (xiv) $1^2, 3^2, 5^2, 7^2, \ldots$

(xv) $1^2, 5^2, 7^2, 73, \ldots$

Sol. (*i*) We have: 2, 4, 8, 16, ...

$$\left.\begin{aligned} T_1 &= 2 \\ T_2 &= 4 \end{aligned}\right\} \Rightarrow T_2 - T_1 = 4 - 2 = 2$$

$$\left.\begin{aligned} T_3 &= 8 \\ T_4 &= 16 \end{aligned}\right\} \Rightarrow T_4 - T_3 = 16 - 8 = 8$$

Since $2 \neq 8$

$\therefore \quad T_2 - T_1 \neq T_4 - T_3$

$\therefore$ The given numbers do **not form an A.P.**

(*ii*) We have: $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

$\therefore \quad T_1 = 2, T_2 = \frac{5}{2}, T_3 = 3, T_4 = \frac{7}{2}$

$$T_2 - T_1 = \frac{5}{2} - 2 = \frac{1}{2}$$

$$T_3 - T_2 = 3 - \frac{5}{2} = \frac{1}{2}$$

$$T_4 - T_3 = \frac{7}{2} - 3 = \frac{1}{2}$$

$\because \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \frac{1}{2} \Rightarrow d = \frac{1}{2}$

$\therefore$ The given numbers **form an A.P.**

$$\therefore \quad T_5 = T_4 + \frac{1}{2} = \frac{7}{2} + \frac{1}{2} = 4$$

$$T_6 = T_5 + \frac{1}{2} = 4 + \frac{1}{2} = \frac{9}{2}$$

$$T_7 = T_6 + \frac{1}{2} = \frac{9}{2} + \frac{1}{2} = 5$$

Thus, $\boldsymbol{d = \frac{1}{2}}$ and $\boldsymbol{T_5 = 4}$, $\boldsymbol{T_6 = \frac{9}{2}}$ and $\boldsymbol{T_7 = 5}$

(*iii*) We have: – 1.2, – 3.2, – 5.2, – 7.2, ...

$\therefore \quad T_1 = -1.2, T_2 = -3.2, T_3 = -5.2, T_4 = -7.2$

$T_2 - T_1 = -3.2 + 1.2 = -2$
$T_3 - T_2 = -5.2 + 3.2 = -2$
$T_4 - T_3 = -7.2 + 5.2 = -2$

$\because \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = -2 \quad \Rightarrow \quad d = -2$

$\therefore$ The given numbers **form an A.P.**

Such that $d = -2$.

Now, $T_5 = T_4 + (-2) = -7.2 + (-2) = -9.2$
$T_6 = T_5 + (-2) = -9.2 + (-2) = -11.2$
$T_7 = T_6 + (-2) = -11.2 + (-2) = -13.2$

Thus, $\boldsymbol{d = -2}$ and $\boldsymbol{T_5 = -9.2, \quad T_6 = -11.2}$ and $\boldsymbol{T_7 = -13.2}$

(*iv*) We have: $-10, \quad -6, \quad -2, \quad 2, ...$

$\therefore \quad T_1 = -10, T_2 = -6, T_3 = -2, T_4 = 2$

$T_2 - T_1 = -6 + 10 = 4$
$T_3 - T_2 = -2 + 6 = 4$
$T_4 - T_3 = 2 + 2 = 4$

$\because \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = 4 \quad \Rightarrow \quad d = 4$

$\therefore$ The given numbers **form an A.P.**

Now, $T_5 = T_4 + 4 = 2 + 4 = 6$
$T_6 = T_5 + 4 = 6 + 4 = 10$
$T_7 = T_6 + 4 = 10 + 4 = 14$

Thus, $\boldsymbol{d = 4}$ and $\boldsymbol{T_5 = 6, \quad T_6 = 10, \quad T_7 = 14}$

(*v*) We have:

$3, \quad 3+\sqrt{2}, \quad 3+2\sqrt{2}, \quad 3+3\sqrt{2}, ...$

$\therefore \quad T_1 = 3, T_2 = 3+\sqrt{2}, T_3 = 3+2\sqrt{2}, T_4 = 3+3\sqrt{2}$

$T_2 - T_1 = 3+\sqrt{2}-3 = \sqrt{2}$

$T_3 - T_2 = 3+2\sqrt{2}-3-\sqrt{2} = \sqrt{2}$

$T_4 - T_3 = 3+3\sqrt{2}-3-2\sqrt{2} = \sqrt{2}$

$\because \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \sqrt{2} \quad \Rightarrow \quad d = \sqrt{2}$

$\Rightarrow$ The given numbers **form an A.P.**

Now, $T_5 = T_4 + \sqrt{2}$

$= 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$T_6 = T_5 + \sqrt{2}$

$= 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$T_7 = T_6 + \sqrt{2}$

$= 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

Thus, $\boldsymbol{d = \sqrt{2}}$ and $\boldsymbol{T_5 = 3+4\sqrt{2}, \quad T_6 = 3+5\sqrt{2}, \quad T_7 = 3+6\sqrt{2}}$.

(*vi*) We have: 0.2, 0.22, 0.222, 0.2222, ...

$\therefore$ $\left.\begin{aligned} T_1 &= 0.2 \\ T_2 &= 0.22 \end{aligned}\right\} \Rightarrow T_2 - T_1 = 0.22 - 0.2 = 0.02$

$\left.\begin{aligned} T_3 &= 0.222 \\ T_4 &= 0.2222 \end{aligned}\right\} \Rightarrow T_4 - T_3 = 0.2222 - 0.222 = 0.0002.$

Since,

$T_2 - T_1 \neq T_4 - T_3$

$\therefore$ The given numbers **do not form an A.P.**

(*vii*) We have: 0, – 4, – 8, – 12, ...

$\therefore$ $T_1 = 0, T_2 = -4, T_3 = -8, T_4 = -12$

$T_2 - T_1 = -4 - 0 = -4$

$T_3 - T_2 = -8 + 4 = -4$

$T_4 - T_3 = -12 + 8 = -4$

$\because$ $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = -4 \Rightarrow d = -4$

$\therefore$ The given numbers **form an A.P.**

Now, $T_5 = T_4 + (-4) = -12 + (-4) = -16$

$T_6 = T_5 + (-4) = -16 + (-4) = -20$

$T_7 = T_6 + (-4) = -20 + (-4) = -24$

Thus, $\boldsymbol{d = -4}$ and $\boldsymbol{T_5 = -16, \; T_6 = -20, \; T_7 = -24}$

(*viii*) We have:

$-\frac{1}{2}, \quad -\frac{1}{2}, \quad -\frac{1}{2}, \quad -\frac{1}{2}, ...$

$\therefore$ $T_1 = T_2 = T_3 = T_4 = -\frac{1}{2}$

$T_2 - T_1 = 0$

$T_3 - T_2 = 0$

$T_4 - T_3 = 0$

$\because$ $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = 0 \Rightarrow d = 0$

$\therefore$ The given number **form an A.P.**

Now, $T_5 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$T_6 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$T_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$

Thus, $\boldsymbol{d = 0}$ and $\boldsymbol{T_5 = -\frac{1}{2}, \; T_6 = -\frac{1}{2}, \; T_7 = -\frac{1}{2}}$

(*ix*) We have: 1, 3, 9, 27, ...

Here, $\left.\begin{aligned} T_1 &= 1 \\ T_2 &= 3 \end{aligned}\right\} \Rightarrow T_2 - T_1 = 3 - 1 = 2$

$\left.\begin{aligned} T_3 &= 9 \\ T_4 &= 27 \end{aligned}\right\} \Rightarrow T_4 - T_3 = 27 - 9 = 18$

$\because$ $T_2 - T_1 \neq T_4 - T_3$

$\therefore$ The given numbers **do not form an A.P.**

(x) We have: $a, \quad 2a, \quad 3a, \quad 4a, \ldots$

$\therefore \quad T_1 = a, T_2 = 2a, T_3 = 3a, T_4 = 4a$

$T_2 - T_1 = 2a - a = a$

$T_3 - T_2 = 3a - 2a = a$

$T_4 - T_3 = 4a - 3a = a$

$\because \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = a \quad \Rightarrow \quad d = a$

$\therefore$ The numbers **form an A.P.**

Now, $T_5 = T_4 + a = 4a + a = 5a$

$T_6 = T_5 + a = 5a + a = 6a$

$T_7 = T_6 + a = 6a + a = 7a$

Thus, $\boldsymbol{d = a}$ and $\boldsymbol{T_5 = 5a, \quad T_6 = 6a, \quad T_7 = 7a}$

(xi) We have: $a, \quad a^2, \quad a^3, \quad a^4, \ldots$

$\therefore \quad \left.\begin{aligned} T_1 &= a \\ T_2 &= a^2 \end{aligned}\right\} \Rightarrow T_2 - T_1 = a^2 - a = a\,[a - 1]$

$\left.\begin{aligned} T_3 &= a^3 \\ T_4 &= a^4 \end{aligned}\right\} \Rightarrow T_4 - T_3 = a^4 - a^3 = a^3\,[a - 1]$

Since,

$T_2 - T_1 \neq T_4 - T_3$

$\therefore$ The given terms are **not in A.P.**

(xii) We have: $\sqrt{2}, \quad \sqrt{8}, \quad \sqrt{18}, \quad \sqrt{32}, \ldots$

$\therefore \quad T_1 = \sqrt{2}, T_2 = \sqrt{8}, T_3 = \sqrt{18}, T_4 = \sqrt{32}$

$T_2 - T_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$T_3 - T_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$T_4 - T_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$\because \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \sqrt{2} \quad \Rightarrow \quad d = \sqrt{2}$

$\therefore$ The given numbers **form an A.P.**

Now, $T_5 = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$T_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$T_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

Thus, $\boldsymbol{d = \sqrt{2}}$ and $\boldsymbol{T_5 = \sqrt{50}, \quad T_6 = \sqrt{72}, \quad T_7 = \sqrt{98}}$

($xiii$) We have: $\sqrt{3}, \quad \sqrt{6}, \quad \sqrt{9}, \quad \sqrt{12}, \ldots$

$\because \quad \left.\begin{aligned} T_1 &= \sqrt{3} \\ T_2 &= \sqrt{6} \end{aligned}\right\} \Rightarrow T_2 - T_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}\left(\sqrt{2} - 1\right)$

and $\left.\begin{aligned} T_3 &= \sqrt{9} \\ T_4 &= \sqrt{12} \end{aligned}\right\} \Rightarrow T_4 - T_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3 = \sqrt{3}\left(2 - \sqrt{3}\right)$

$\therefore \quad T_2 - T_1 \neq T_4 - T_3$

$\Rightarrow$ The given terms **do not form an A.P.**

(*xiv*) We have: 1^2, 3^2, 5^2, 7^2, ...

$$\therefore \quad \left.\begin{aligned} T_1 &= 1^2 = 1 \\ T_2 &= 3^2 = 9 \end{aligned}\right\} \Rightarrow \quad T_2 - T_1 = 9 - 1 = 8$$

$$\left.\begin{aligned} T_3 &= 5^2 = 25 \\ T_4 &= 7^2 = 49 \end{aligned}\right\} \Rightarrow \quad T_4 - T_3 = 49 - 25 = 24$$

$\because \quad T_2 - T_1 \neq T_4 - T_3$

$\therefore$ The given terms **do not form an A.P.**

(*xv*) We have: 1^2, 5^2, 7^2, 73, ...

$\therefore \quad T_1 = 1^2 = 1,\ T_2 = 5^2 = 25,\ T_3 = 7^2 = 49,\ T_4 = 73$

$T_2 - T_1 = 25 - 1 = 24$

$T_3 - T_2 = 49 - 25 = 24$

$T_4 - T_3 = 73 - 49 = 24$

$\because \quad T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = 24 \quad \Rightarrow \quad d = 24$

$\therefore$ The numbers **form an A.P.**

Now, $T_5 = T_4 + 24 = 73 + 24 = 97$

$T_6 = T_5 + 24 = 97 + 24 = 121$

$T_7 = T_6 + 24 = 121 + 24 = 145$

Thus, $\boldsymbol{d = 24}$ and $\boldsymbol{T_5 = 97,\ T_6 = 121,\ T_7 = 145}$

- ***n*th Term of an A.P.**

The nth term T_n of the A.P. with first term 'a' and common difference 'd' is given by

$$T_n = a + (n - 1)\,d$$

'T_n' is also called the general term of the A.P. If there are 'm' terms in the A.P., then 'T_m' represents the last term which is generally denoted by 'l'.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 5.2

Q. 1. *Fill in the blanks in the following table, given that 'a' is the first term, 'd' the common difference and a_n the nth term of the A.P.:*

	a	*d*	*n*	a_n
(i)	*7*	*3*	*8*	…
(ii)	*– 18*	…	*10*	*0*
(iii)	…	*– 3*	*18*	*– 5*
(iv)	*– 18.9*	*2.5*	…	*3.6*
(v)	*3.5*	*0*	*105*	…

Sol. (*i*)

$$a_n = a + (n - 1)\,d$$

$$\Rightarrow \quad a_8 = 7 + (8 - 1)\,3$$

$$= 7 + 7 \times 3$$

$$= 7 + 21$$

$$\Rightarrow \quad a_8 = \mathbf{28}$$

(*ii*) $a_n = a + (n-1)d$

$\Rightarrow \quad a_{10} = -18 + (10-1)d$

$\Rightarrow \quad 0 = -18 + 9d$

$\Rightarrow \quad 9d = 18 \Rightarrow d = \dfrac{18}{9} = 2$

$\therefore \quad \mathbf{d = 2}$

(*iii*) $a_n = a + (n-1)d$

$\Rightarrow \quad -5 = a + (18-1) \times (-3)$

$\Rightarrow \quad -5 = a + 17 \times (-3)$

$\Rightarrow \quad -5 = a - 51$

$\Rightarrow \quad a = -5 + 51 = 46$

Thus, $\mathbf{a = 46}$

(*iv*) $a_n = a + (n-1)d$

$\Rightarrow \quad 3.6 = -18.9 + (n-1) \times 2.5$

$\Rightarrow \quad (n-1) \times 2.5 = 3.6 + 18.9$

$\Rightarrow \quad (n-1) \times 2.5 = 22.5$

$\Rightarrow \quad n - 1 = \dfrac{22.5}{2.5} = 9$

$\Rightarrow \quad n = 9 + 1 = 10$

Thus, $\mathbf{n = 10}$

(*v*) $a_n = a + (n-1)d$

$\Rightarrow \quad a_n = 3.5 + (105-1) \times 0$

$\Rightarrow \quad a_n = 3.5 + 104 \times 0$

$\Rightarrow \quad a_n = 3.5 + 0 = 3.5$

Thus, $\mathbf{a_n = 3.5}$

Q. 2. *Choose the correct choice in the following and justify:*

(i) 30th term of the A.P.: 10, 7, 4, ..., is

(A) 97 *(B) 77* *(C) – 77* *(D) – 87*

(ii) 11th term of the A.P.: $-3, -\frac{1}{2}, 2$, *...., is*

(A) 28 *(B) 22* *(C) – 38* *(D)* $-48\frac{1}{2}$

Sol. (*i*) Here, $a = 10, \quad n = 30$

$\because \quad T_n = a + (n-1)d$ and $d = 7 - 10 = -3$

$\therefore \quad T_{30} = 10 + (30-1) \times (-3)$

$\Rightarrow \quad T_{30} = 10 + 29 \times (-3)$

$\Rightarrow \quad T_{30} = 10 - 87 = -77$

Thus, the correct choice is **(*C*) – 77.**

(*ii*) Here, $a = -3$, $n = 11$ and $d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}$

$\therefore \quad T_n = a + (n-1)d$

$\Rightarrow \quad T_{11} = -3 + (11-1) \times \frac{5}{2}$

$\Rightarrow \quad T_{11} = -3 + 25 = 22$

Thus, the correct choice is **(*B*) 22.**

Q. 3. *In the following A.Ps., find the missing terms in the boxes:*

(*i*) 2, $\square$, 26

(*ii*) $\square$, 13, $\square$, 3

(*iii*) 5, $\square$, $\square$, $9\frac{1}{2}$

(*iv*) – 4, $\square$, $\square$, $\square$, $\square$, 6

(*v*) $\square$, 38, $\square$, $\square$, $\square$, – 22

Sol. (*i*) Here, $a = 2$, $T_3 = 26$

Let common difference = d

$\therefore \quad T_n = a + (n-1)d$

$\Rightarrow \quad T_3 = 2 + (3-1)d$

$\Rightarrow \quad 26 = 2 + 2d$

$\Rightarrow \quad 2d = 26 - 2 = 24$

$\Rightarrow \quad d = \frac{24}{2} = 12$

$\therefore$ The missing term $= a + d$

$= 2 + 12 = \boxed{\mathbf{14}}$

(*ii*) Let the first term = a and common difference = d

Here, $T_2 = 13$ and $T_4 = 3$

$T_2 = a + d = 13$

$T_4 = a + 3d = 3$

$\therefore \quad T_4 - T_2 = (a + 3d) - (a + d) = 3 - 13$

$\Rightarrow \quad 2d = -10$

$\Rightarrow \quad d = \frac{-10}{2} = -5$

Now, $a + d = 13 \Rightarrow a + (-5) = 13$

$\Rightarrow \quad a = 13 + 5 = 18$

Thus, missing terms are a and $a + 2d$ or 18 and 18 + (– 10) = 8

i.e., $\quad T_1 = \boxed{\mathbf{18}}$ and $T_3 = \boxed{\mathbf{8}}$

(*iii*) Here, $a = 5$ and $T_4 = 9\frac{1}{2}$

since, $T_4 = a + 3d$

$\Rightarrow \quad 9\frac{1}{2} = 5 + 3d$

$\Rightarrow \quad 3d = 9\frac{1}{2} - 5 = 4\frac{1}{2}$

$\Rightarrow \quad d = 4\frac{1}{2} \div 3 = \frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$

$\therefore$ The missing terms are:

$$T_2 = a + d = 5 + \frac{3}{2} = \boxed{\mathbf{6\frac{1}{2}}}$$

$$T_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = \boxed{\mathbf{8}}$$

(*iv*) Here, $a = -4$ and $T_6 = 6$

$\because \quad T_n = a + (n - 1)\, d$

$\therefore \quad T_6 = -4 + (6 - 1)\, d$

$\Rightarrow \quad 6 = -4 + 5d$

$\Rightarrow \quad 5d = 6 + 4 = 10$

$\Rightarrow \quad d = 10 \div 5 = 2$

$\therefore \quad T_2 = a + d = -4 + 2 = -2$

$T_3 = a + 2d = -4 + 2\,(2) = 0$

$T_4 = a + 3d = -4 + 3\,(2) = 2$

$T_5 = a + 4d = -4 + 4\,(2) = 4$

$\therefore$ The missing terms are $\boxed{\mathbf{-2}}$, $\boxed{\mathbf{0}}$, $\boxed{\mathbf{2}}$, $\boxed{\mathbf{4}}$

(*v*) Here, $T_2 = 38$ and $T_6 = -22$

$\therefore \quad T_2 = a + d = 38$

$T_6 = a + 5d = -22$

$\Rightarrow \quad T_6 - T_2 = a + 5d - (a + d) = -22 - 38$

$\Rightarrow \quad 4d = -60$

$\Rightarrow \quad d = \frac{-60}{4} = -15$

$\therefore \quad a + d = 38 \Rightarrow a + (-15) = 38$

$\Rightarrow \quad a = 38 + 15 = 53$

Now, $T_3 = a + 2d = 53 + 2\,(-15) = 53 - 30 = 23$

$T_4 = a + 3d = 53 + 3\,(-15) = 53 - 45 = 8$

$T_5 = a + 4d = 53 + 4\,(-15) = 53 - 60 = -7$

Thus, the missing terms are $\boxed{\mathbf{53}}$, $\boxed{\mathbf{23}}$, $\boxed{\mathbf{8}}$, $\boxed{\mathbf{-7}}$

Q. 4. *Which term of the A.P.: 3, 8, 13, 18, ..., is 78?*

Sol. Let the nth term = 78

Here, $a = 3, \Rightarrow T_1 = 3$ and $T_2 = 8$

$\therefore \quad d = T_2 - T_1 = 8 - 3 = 5$

Now, $T_n = a + (n - 1)\, d$

$\Rightarrow \quad 78 = 3 + (n - 1) \times 5$

$\Rightarrow \quad 78 - 3 = (n - 1) \times 5$

$\Rightarrow \quad 75 = (n - 1) \times 5$

$\Rightarrow \quad (n - 1) = 75 \div 5 = 15$

$\Rightarrow \quad n = 15 + 1 = 16$

Thus, 78 is the **16**th term of the given A.P.

Q. 5. *Find the number of terms in each of the following A.Ps.:*

(i) 7, 13, 19, ..., 205 *(ii) 18, $15\frac{1}{2}$, 13, ..., – 47*

Sol. (*i*) Here, $a = 7$

$$d = 13 - 7 = 6$$

Let the number of terms be n

$\therefore \quad T_n = 205$

Now, $T_n = a + (n - 1) \times d$

$\Rightarrow \quad 7 + (n - 1) \times 6 = 205$

$\Rightarrow \quad (n - 1) \times 6 = 205 - 7 = 198$

$\Rightarrow \quad n - 1 = \frac{198}{6} = 33$

$\therefore \quad n = 33 + 1 = 34$

Thus, the required number of terms is **34.**

(*ii*) Here, $a = 18$

$$d = 15\frac{1}{2} - 18 = -2\frac{1}{2}$$

Let the nth term = – 47

$\therefore \quad T_n = a + (n - 1)\, d$

$\Rightarrow \quad -47 = 18 + (n - 1) \times \left(-2\frac{1}{2}\right)$

$\Rightarrow \quad -47 - 18 = (n - 1) \times \left(\frac{-5}{2}\right)$

$\Rightarrow \quad -65 = (n - 1) \times \left(\frac{-5}{2}\right)$

$\Rightarrow \quad n - 1 = -65 \times \left(\frac{-2}{5}\right)$

$\Rightarrow \quad n - 1 = (-13) \times (-2) = 26$

$\Rightarrow \quad n = 26 + 1 = 27$

Thus, the required number of terms is **27.**

Q. 6. *Check whether – 150 is a term of the A.P.: 11, 8, 5, 2 ...*

Sol. For the given *A.P.*, we have

$$a = 11$$

$$d = 8 - 11 = -3$$

Let – 150 is the *n*th term of the given A.P.

$$\therefore \quad T_n = a + (n-1)d$$

$$\Rightarrow \quad -150 = 11 + (n-1) \times (-3)$$

$$\Rightarrow \quad -150 - 11 = (n-1) \times (-3)$$

$$\Rightarrow \quad -161 = (n-1) \times (-3)$$

$$\Rightarrow \quad n - 1 = \frac{-161}{-3} = \frac{161}{3}$$

$$\Rightarrow \quad n = \frac{161}{3} + 1 = \frac{164}{3} = 54\frac{2}{3}$$

But *n* should be a positive integer.

Thus, – 150 is **not a term** of the given A.P.

Q. 7. *Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73.*

Sol. Here, $T_{31} = ?$

$$T_{11} = 38$$

$$T_{16} = 73$$

If the first term = *a* and the common difference = *d.*

Then,

$$a + (11-1)d = 38$$

$$\Rightarrow \quad a + 10 - d = 38 \qquad \text{...(1)}$$

$$\text{and} \quad a + (16-1)d = 73$$

$$\Rightarrow \quad a + 15d = 73 \qquad \text{...(2)}$$

Subtracting (1) from (2), we get

$$(a + 15d) - (a + 10 - d) = 73 - 38$$

$$\Rightarrow \quad 5d = 35$$

$$\Rightarrow \quad d = \frac{35}{5} = 7$$

From (1),

$$a + 10(7) = 38$$

$$\Rightarrow \quad a + 70 = 38$$

$$\Rightarrow \quad a = 38 - 70 = -32$$

$$\therefore \quad T_{31} = -32 + (31-1) \times 7$$

$$\Rightarrow \quad T_{31} = -32 + 30 \times 7$$

$$\Rightarrow \quad T_{31} = -32 + 210$$

$$\Rightarrow \quad T_{31} = 178$$

Thus, the 31st term is **178.**

Q. 8. *An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.*

Sol. Here, $n = 50$

$T_3 = 12$

$T_n = 106 \Rightarrow T_{50} = 106$

If first term = a and the common difference = d

$\therefore \quad T_3 = a + 2d = 12$ ...(1)

$T_{50} = a + 49d = 106$ ...(2)

$\Rightarrow T_{50} - T_3 \Rightarrow a + 49d - (a + 2d) = 106 - 12$

$\Rightarrow \quad 47d = 94$

$\Rightarrow \quad d = \frac{94}{47} = 2$

From (1), we have

$a + 2d = 12 \Rightarrow a + 2(2) = 12$

$\Rightarrow a = 12 - 4 = 8$

Now, $T_{29} = a + (29 - 1)d$

$= 8 + (28) \times 2$

$= 8 + 56 = 64$

Thus, the 29th term is **64.**

Q. 9. *If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively, which term of this A.P. is zero ?*

Sol. Here, $T_3 = 4$ and $T_9 = -8$

$\therefore$ Using $T_n = a + (n - 1)d$

$\Rightarrow \quad T_3 = a + 2d = 4$ ...(1)

$T_9 = a + 8d = -8$ ...(2)

Subtracting (1) from (2) we get

$(a + 8d) - (a + 2d) = -8 - 4$

$\Rightarrow \quad 6d = -12$

$\Rightarrow \quad d = \frac{-12}{6} = -2$

Now, from (1), we have:

$a + 2d = 4$

$\Rightarrow \quad a + 2(-2) = 4$

$\Rightarrow \quad a - 4 = 4$

$\Rightarrow \quad a = 4 + 4 = 8$

Let the nth term of the A.P. be 0.

$\therefore \quad T_n = a + (n - 1)d = 0$

$\Rightarrow \quad 8 + (n - 1) \times (-2) = 0$

$\Rightarrow \quad (n - 1) \times -2 = -8$

$\Rightarrow \quad n - 1 = \frac{-8}{-2} = 4$

$\Rightarrow \quad n = 4 + 1 = 5$

Thus, the **5**th **term** of the A.P. is 0.

Q. 10. *The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.*

Sol. Let 'a' be the first term and 'd' be the common difference of the given A.P.

Now, using $T_n = a + (n - 1)\, d$

$$T_{17} = a + 16d$$
$$T_{10} = a + 9d$$

According to the condition,

$$T_n + 7 = T_{17}$$
$$\Rightarrow \quad (a + 9d) + 7 = a + 16d$$
$$\Rightarrow \quad a + 9d - a - 16d = -7$$
$$\Rightarrow \quad -7d = -7 \quad \Rightarrow \quad d = 1$$

Thus, the common difference is 1.

Q. 11. *Which term of the A.P.: 3, 15, 27, 39, ... will be 132 more than its 54th term?*

Sol. Here, $a = 3$

$$d = 15 - 3 = 12$$

Using $T_n = a + (n - 1)\, d$, we get

$$T_{54} = a + 53d$$
$$= 3 + 53 \times 12$$
$$= 3 + 636 = 639$$

Let a_n be 132 more than its 54th term.

$$\therefore \quad a_n = T_{54} + 132$$
$$\Rightarrow \quad a_n = 639 + 132 = 771$$

Now $a_n = a + (n - 1)\, d = 771$

$$\Rightarrow \quad 3 + (n - 1) \times 12 = 771$$
$$\Rightarrow \quad (n - 1) \times 12 = 771 - 3 = 768$$
$$\Rightarrow \quad (n - 1) = \frac{768}{12} = 64$$
$$\Rightarrow \quad n = 64 + 1 = 65$$

Thus, 132 more than 54th term is the **65**th **term.**

Q. 12. *Two A.Ps. have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?*

Sol. Let for the 1st A.P., the first term = a

$$\therefore \quad T_{100} = a + 99d$$

And for the 2nd A.P., the first term = a'

$$\therefore \quad T'_{100} = a' + 99d$$

According to the condition, we have:

$$T_{100} - T'_{100} = 100$$
$$\Rightarrow \quad a + 99d - (a' + 99d) = 100$$
$$\Rightarrow \quad a - a' = 100$$

Let, $T_{1000} - T'_{1000} = x$

$$\therefore \quad a + 999d - (a' + 999d) = x$$
$$\Rightarrow \quad a - a' = x \quad \Rightarrow \quad x = 100$$

$\therefore$ The difference between the 1000th terms is **100.**

Q. 13. *How many three-digit numbers are divisible by 7?*

Ans. The first three digit number divisible by 7 is 105.

The last such three digit number is 994.

$\therefore$ The A.P. is 105, 112, 119, ..., 994

Here, $a = 105$ and $d = 7$

Let n be the required number of terms.

$$\therefore \quad T_n = a + (n-1)d$$
$$\Rightarrow \quad 994 = 105 + (n-1) \times 7$$
$$\Rightarrow \quad (n-1) \times 7 = 994 - 105 = 889$$
$$\Rightarrow \quad (n-1) = \frac{889}{7} = 127$$
$$\Rightarrow \quad n = 127 + 1 = 128$$

Thus, **128** numbers of 3-digit are divisible by 7.

Q. 14. *How many multiples of 4 lie between 10 and 250?*

Sol. $\because$ The first multiple of 4 beyond 10 is 12.

The multiple of 4 just below 250 is 248.

$\therefore$ The A.P. is given by:

12, 16, 20, ..., 248

Here, $a = 12$ and $d = 4$

Let the number of terms = n

$\therefore$ Using $T_n = a + (n-1)d$, we get

$$\therefore \quad T_n = 12 + (n-1) \times 4$$
$$\Rightarrow \quad 248 = 12 + (n-1) \times 4$$
$$\Rightarrow \quad (n-1) \times 4 = 248 - 12 = 236$$
$$\Rightarrow \quad n - 1 = \frac{236}{4} = 59$$
$$\Rightarrow \quad n = 59 + 1 = 60$$

Thus, the required number of terms = 60.

Q. 15. *For what value of n, are the nth terms of two A.Ps.: 63, 65, 67, ... and 3, 10, 17, ... equal?*

Sol. For the 1st **A.P.**

$$\because \quad a = 63 \text{ and } d = 65 - 63 = 2$$
$$\therefore \quad T_n = a + (n-1)d$$
$$\Rightarrow \quad T_n = 63 + (n-1) \times 2$$

For the 2nd **A.P.**

$$\because \quad a = 3 \text{ and } d = 10 - 3 = 7$$
$$\therefore \quad T_n = a + (n-1)d$$
$$\Rightarrow \quad T_n = 3 + (n-1) \times 7$$

Now, according to the condition,

$$3 + (n-1) \times 7 = 63 + (n-1) \times 2$$
$$\Rightarrow \quad (n-1) \times 7 - (n-1) \times 2 = 63 - 3$$

$\Rightarrow \quad 7n - 7 - 2n + 2 = 60$

$\Rightarrow \quad 5n - 5 = 60$

$\Rightarrow \quad 5n = 60 + 5 = 65$

$\Rightarrow \quad n = \frac{65}{5} = 13$

Thus, the **13**th **terms** of the two given A.Ps. are equal.

Q. 16. *Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.*

Sol. Let the first term = a and the common difference = d.

$\therefore$ Using $T_n = a + (n - 1)\, d$, we have:

$T_3 = a + 2d$

$\Rightarrow a + 2d = 16$ ...(1)

And $T_7 = a + 6d$, $T_5 = a + 4d$

According to the condition,

$T_7 - T_5 = 12$

$\Rightarrow \quad (a + 6d) - (a + 4d) = 12$

$\Rightarrow \quad a + 6d - a - 4d = 12$

$\Rightarrow \quad 2d = 12$

$\Rightarrow \quad d = \frac{12}{2} = 6$ ...(2)

Now, from (1) and (2), we have:

$a + 2\,(6) = 16$

$\Rightarrow \quad a + 12 = 16$

$\Rightarrow \quad a = 16 - 12 = 4$

$\therefore$ The required A.P. is

4, [4 + 6], [4 + 2 (6)], [4 + 3 (6)], ...

or **4, 10, 16, 22, ...**

Q. 17. *Find the 20th term from the last term of the A.P.: 3, 8, 13, ..., 253.*

Sol. We have, the last term $l = 253$

Here, $d = 8 - 3 = 5$

Since, the nth term before the last term is given by $l - (n - 1)\, d$,

$\therefore$ We have

20th term from the end $= l - (20 - 1) \times 5$

$= 253 - 19 \times 5$

$= 253 - 95 =$ **158**

Q. 18. *The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.*

Sol. Let the first term = a

And the common difference = d

$\therefore$ Using $T_n = a + (n - 1)\, d$,

$T_4 + T_8 = 24$

$\Rightarrow \quad (a + 3d) + (a + 7d) = 24$

$\Rightarrow \quad 2a + 10d = 24$

$\Rightarrow \quad a + 5d = 12 \quad ...(1)$

And $\quad T_6 + T_{10} = 44$

$\Rightarrow \quad (a + 5d) + (a + 9d) = 44$

$\Rightarrow \quad 2a + 14d = 44$

$\Rightarrow \quad a + 7d = 22 \quad ...(2)$

Now, subtracting (1) from (2), we get

$(a + 7d) - (a + 5d) = 22 - 12$

$\Rightarrow \quad 2d = 10$

$\Rightarrow \quad d = \frac{10}{2} = 5$

$\therefore$ From (1), $\quad a + 5 \times 5 = 12$

$\Rightarrow a + 25 = 12$

$\Rightarrow \quad a = 12 - 25 = -13$

Now, the first three terms of the A.P. are given by:

$a, (a + d), (a + 2d)$

or $-13, (-13 + 5), [-13 + 2(5)]$

or $\mathbf{-13, -8, -3}$

Q. 19. *Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?*

Sol. Here, $\quad a$ = ₹ 5000 $\quad$ and $\quad d$ = ₹ 200

Say, in the nth year he gets ₹ 7000.

$\therefore$ Using $\quad T_n = a + (n - 1)d,$ $\quad$ we get

$7000 = 5000 + (n - 1) \times 200$

$\Rightarrow (n - 1) \times 200 = 7000 - 5000 = 2000$

$\Rightarrow \quad n - 1 = \frac{2000}{200} = 10$

$\Rightarrow \quad n = 10 + 1 = 11$

Thus, his income becomes ₹ 7000 in **11 years.**

Q. 20. *Ramkali saved ₹ 5 in the first week of a year and then increased weekly savings by ₹ 1.75. If in the nth week, her weekly savings become ₹ 20.75, find n.*

Sol. Here, $\quad a$ = ₹ 5 $\quad$ and $\quad d$ = ₹ 1.75

$\because$ In the nth week her savings become ₹ 20.75.

$\therefore \quad T_n$ = ₹ 20.75

$\therefore$ Using $\quad T_n = a + (n - 1)d,$ $\quad$ we have

$20.75 = 5 + (n - 1) \times (1.75)$

$\Rightarrow \quad (n - 1) \times 1.75 = 20.75 - 5$

$\Rightarrow \quad (n - 1) \times 1.75 = 15.75$

$\Rightarrow \quad n - 1 = \frac{15.75}{1.75} = 9$

$\Rightarrow \quad n = 9 + 1 = 10$

Thus, the required number of years = **10.**

- **Sum of First n Terms of an A.P.**

  (*i*) If the first term of an A.P. is 'a' and the common difference is 'd' then the sum of its first n terms is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

  (*ii*) If the last term of the A.P. is l then

$$S_n = \frac{n}{2}(a + l)$$

Remember,

The sum of first n positive integers is given by:

$$S_n = \frac{n(n+1)}{2}$$

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 5.3

Q. 1. *Find the sum of the following A.Ps.:*

(i) 2, 7, 12, ..., to 10 terms. *(ii) – 37, – 33, – 29, ..., to 12 terms.*

(iii) 0.6, 1.7, 2.8, ..., to 100 terms. *(iv)* $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, *..., to 11 terms.*

Sol. (*i*) Here,

$$a = 2$$
$$d = 7 - 2 = 5$$
$$n = 10$$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 5]$

$\Rightarrow$ $S_{10} = 5[4 + 9 \times 5]$

$\Rightarrow$ $S_{10} = 5[49] = 245$

Thus, the sum of first 10 terms is **245.**

(*ii*) We have:

$$a = -37$$
$$d = -33 - (-37) = 4$$
$$n = 12$$

$\therefore$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_{12} = \frac{12}{2}[2(-37) + (12-1) \times 4]$

$= 6[-74 + 11 \times 4]$

$= 6[-74 + 44]$

$= 6 \times [-30] = -180$

Thus, sum of first 12 terms = **–180.**

(*iii*) Here, $a = 0.6$

$d = 1.7 - 0.6 = 1.1$

$n = 100$

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{100} = \frac{100}{2}[2(0.6) + (100-1) \times 1.1]$$

$$= 50[1.2 + 99 \times 1.1]$$

$$= 50[1.2 + 108.9]$$

$$= 50[110.1]$$

$$= 5505$$

Thus, the required sum of first 100 terms is **5505.**

(*iv*) Here, $a = \frac{1}{15}$

$$d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$$

$n = 11$

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[\left(2 \times \frac{1}{15}\right) + (11-1) \times \frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \left(10 \times \frac{1}{60}\right)\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$$

$$= \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \frac{11}{2} \times \frac{9}{30} = \frac{99}{60} = \frac{33}{20}$$

Thus, the required sum of first 11 terms = $\mathbf{\frac{33}{20}}$.

Q. 2. *Find the sums given below:*

(*i*) $7 + 10\frac{1}{2} + 14 + ... + 84$

(*ii*) $34 + 32 + 30 + ... + 10$

(*iii*) $-5 + (-8) + (-11) + ... + (-230)$

Sol. (*i*) Here, $a = 7$

$$d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$$

$l = 84$

Let n be the number of terms

$$\therefore \quad T_n = a + (n-1)\,d$$

$$\Rightarrow \quad 84 = 7 + (n-1) \times \frac{7}{2}$$

$$\Rightarrow \quad (n-1) \times \frac{7}{2} = 84 - 7 = 77$$

$$\Rightarrow \quad n - 1 = 77 \times \frac{2}{7} = 22$$

$$\Rightarrow \quad n = 22 + 1 = 23$$

Now, $$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow \quad S_{23} = \frac{23}{2}(7 + 84)$$

$$= \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$$

Thus, the required sum = $\mathbf{1046\frac{1}{2}}$.

(*ii*) Here, $a = 34$

$$d = 32 - 34 = -2$$

$$l = 10$$

Let the number of terms be n

$$\therefore \quad T_n = 10$$

Now $$T_n = a + (n-1)\,d$$

$$\Rightarrow \quad 10 = 34 + (n-1) \times (-2)$$

$$\Rightarrow \quad (n-1) \times (-2) = 10 - 34 = -24$$

$$\Rightarrow \quad n - 1 = \frac{-24}{-2} = 12$$

$$\Rightarrow \quad n = 13$$

Now, $$S_n = \frac{n}{2}\,[2a + (n-1)\,d]$$

$$\Rightarrow \quad S_{13} = \frac{13}{2}\,[2 \times 34 + (13-1) \times (-2)]$$

$$= \frac{13}{2}\,[68 + 12 \times (-2)]$$

$$= \frac{13}{2}\,[68 - 24]$$

$$= \frac{13}{2}[44]$$

$$= 13 \times 22 = 286$$

OR

$$S_{13} = \frac{n}{2}\,(a + l)$$

$$= \frac{13}{2}\,(34 + 10)$$

$$= \frac{13}{2} \times 44 = 13 \times 22 = 286$$

Thus, the required sum is **286.**

(*iii*) Here, $a = -5$

$d = -8 - (-5) = -3$

$l = -230$

Let n be the number of terms.

$\therefore \quad T_n = -230$

$\Rightarrow \quad -230 = -5 + (n-1) \times (-3)$

$\Rightarrow \quad (n-1) \times (-3) = -230 + 5 = -225$

$\Rightarrow \quad n - 1 = \dfrac{-225}{-3} = 75$

$\Rightarrow \quad n = 75 + 1 = 76$

Now, $S_{76} = \dfrac{76}{2}\,[(-5) + (-230)]$

$= 38 \times (-235)$

$= -8930$

$\therefore$ The required sum = **– 8930.**

Q. 3. *In an A.P.:*

(*i*) *given* $a = 5$, $d = 3$, $a_n = 50$, *find* n *and* S_n.

(*ii*) *given* $a = 7$, $a_{13} = 35$, *find* d *and* S_{13}.

(*iii*) *given* $a_{12} = 37$, $d = 3$, *find* a *and* S_{12}.

(*iv*) *given* $a_3 = 15$, $S_{10} = 125$, *find* d *and* a_{10}.

(*v*) *given* $d = 5$, $S_9 = 75$, *find* a *and* a_9.

(*vi*) *given* $a = 2$, $d = 8$, $S_n = 90$, *find* n *and* a_n.

(*vii*) *given* $a = 8$, $a_n = 62$, $S_n = 210$, *find* n *and* d.

(*viii*) *given* $a_n = 4$, $d = 2$, $S_n = -14$, *find* n *and* a.

(*ix*) *given* $a = 3$, $n = 8$, $S = 192$, *find* d.

(*x*) *given* $l = 28$, $S = 144$, *and there are total 9 terms. Find* a.

Sol. (*i*) Here, $a = 5,\ d = 3$ and $a_n = 50 = l$

$\because \quad a_n = a + (n-1)\,d$

$\therefore \quad 50 = 5 + (n-1) \times 3$

$\Rightarrow \quad 50 - 5 = (n-1) \times 3$

$\Rightarrow \quad (n-1) \times 3 = 45$

$\Rightarrow \quad (n-1) = \dfrac{45}{3} = 15$

$\Rightarrow \quad n = 15 + 1 = 16$

Now $S_n = \dfrac{n}{2}\,(a + l)$

$= \dfrac{16}{2}\,(5 + 50)$

$= 8\,(55) = 440$

Thus, $\boldsymbol{n = 16}$ and $\boldsymbol{S_n = 440}$

(*ii*) Here, $a = 7$ and $a_{13} = 35 = l$

$\therefore \quad a_n = a + (n-1)\,d$

$\Rightarrow \quad 35 = 7 + (13-1)\,d$

$\Rightarrow \quad 35 - 7 = 12d$

$\Rightarrow \quad 28 = 12d$

$\Rightarrow \quad d = \frac{28}{12} = \frac{7}{3}$

Now, using

$$S_n = \frac{n}{2}(a + l)$$

$$S_{13} = \frac{13}{2}(7 + 35)$$

$$= \frac{13}{2} \times 42$$

$$= 13 \times 21 = 273$$

$S_n = \mathbf{273}$ and $d = \mathbf{\frac{7}{3}}$

(*iii*) Here, $a_{12} = 37 = l$ and $d = 3$

Let the first term of the *A.P.* be '*a*'.

Now $\quad a_{12} = a + (12-1)\,d$

$\Rightarrow \quad 37 = a + 11d$

$\Rightarrow \quad 37 = a + 11 \times 3$

$\Rightarrow \quad 37 = a + 33$

$\Rightarrow \quad a = 37 - 33 = 4$

Now, $\quad S_n = \frac{n}{2}(a + l)$

$\Rightarrow \quad S_{12} = \frac{12}{2}(4 + 37)$

$\Rightarrow \quad S_{12} = 6 \times (41) = 246$

Thus, $\boldsymbol{a = 4}$ and $\boldsymbol{S_{12} = 246}$

(*iv*) Here, $a_3 = 15 = l$

$S_{10} = 125$

Let first term of the *A.P.* be '*a*' and the common difference = *d*

$\therefore \quad a_3 = a + 2d$

$\Rightarrow \quad a + 2d = 15$ ...(1)

Again $\quad S_n = \frac{n}{2}[2a + (n-1)\,d]$

$\Rightarrow \quad S_{10} = \frac{10}{2}[2a + (10-1)\,d]$

$\Rightarrow \quad 125 = 5\,[2a + 9d]$

$\Rightarrow \quad 2a + 9d = \dfrac{125}{5} = 25$

$\Rightarrow \quad 2a + 9d = 25$ ...(2)

Multiplying (1) by 2 and subtracting (2) from it, we get

$2\ [a + 2d = 15] - [2a + 9d = 25]$

$\Rightarrow 2a + 4d - 2a - 9d = 30 - 25$

$\Rightarrow \quad -5d = 5$

$\Rightarrow \quad d = \dfrac{5}{-5} = -1$

$\therefore$ From (1), $a + 2(-1) = 15 \quad \Rightarrow \quad a = 15 + 2 \quad \Rightarrow \quad a = 17$

Now, $\quad a_{10} = a + (10 - 1)\ d$

$= 17 + 9 \times (-1)$

$= 17 - 9 = 8$

Thus, $\quad \boldsymbol{d = -1}$ and $\boldsymbol{a_{10} = 8}$

(*v*) Here, $\quad d = 5, \quad S_9 = 75$

Let the first term of the *A.P.* is '*a*'.

$\therefore \quad S_9 = \dfrac{9}{2}\ [2a + (9 - 1) \times 5]$

$\Rightarrow \quad 75 = \dfrac{9}{2}\ [2a + 40]$

$\Rightarrow \quad 75 \times \dfrac{2}{9} = 2a + 40$

$\Rightarrow \quad \dfrac{50}{3} = 2a + 40$

$\Rightarrow \quad 2a = \dfrac{50}{3} - 40 = \dfrac{-70}{3}$

$\Rightarrow \quad a = \dfrac{-70}{3} \times \dfrac{1}{2} = \dfrac{-35}{3}$

Now, $\quad a_9 = a + (9 - 1)\ d$

$= \dfrac{-35}{3} + (8 \times 5)$

$= \dfrac{-35}{3} + 40$

$= \dfrac{-35 + 120}{3} = \dfrac{85}{3}$

Thus, $\quad \boldsymbol{a = \dfrac{-35}{3}}$ and $\boldsymbol{a_9 = \dfrac{85}{3}}$

(*vi*) Here, $\quad a = 2, \quad d = 8$ and $S_n = 90$

$\because \quad S_n = \dfrac{n}{2}\ [2a + (n - 1)\ d]$

$\therefore \quad 90 = \frac{n}{2}[2 \times 2 + (n-1) \times 8]$

$\Rightarrow \quad 90 \times 2 = 4n + n(n-1) \times 8$

$\Rightarrow \quad 180 = 4n + 8n^2 - 8n$

$\Rightarrow \quad 180 = 8n^2 - 4n$

$\Rightarrow \quad 45 = 2n^2 - n$

$\Rightarrow \quad 2n^2 - n - 45 = 0$

$\Rightarrow \quad 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow \quad 2n(n-5) + 9(n-5) = 0$

$\Rightarrow \quad (2n+9)(n-5) = 0$

$\therefore$ Either $\quad 2n + 9 = 0 \Rightarrow n = -\frac{9}{2}$

or $\quad n - 5 = 0 \Rightarrow n = 5$

But $n = -\frac{9}{2}$ is not required.

$\therefore \quad n = 5$

Now, $\quad a_n = a + (n-1)d$

$\Rightarrow \quad a_5 = 2 + (5-1) \times 8$

$= 2 + 32 = 34$

Thus, $\quad \boldsymbol{n = 5}$ **and** $\boldsymbol{a_5 = 34}$**.**

(*vii*) Here, $\quad a = 8, \; a_n = 62 = l$ and $S_n = 210$

Let the common difference $= d$

Now, $\quad S_n = 210$

$\Rightarrow \quad 210 = \frac{n}{2}(a + l)$

$\Rightarrow \quad 210 = \frac{n}{2}(8 + 62) = \frac{n}{2} \times 70 = 35n$

$\therefore \quad n = \frac{210}{35} = 6$

Again $\quad a_n = a + (n-1)d$

$\Rightarrow \quad 62 = 8 + (6-1) \times d$

$\Rightarrow \quad 62 - 8 = 5d$

$\Rightarrow \quad 54 = 5d \Rightarrow d = \frac{54}{5}$

Thus, $\quad \boldsymbol{n = 6}$ **and** $\boldsymbol{d = \frac{54}{5}}$**.**

(*viii*) Here, $\quad a_n = 4, \; d = 2$ and $S_n = -14$

Let the first term be '*a*'.

$\because \quad a_n = 4$

$\therefore \quad a + (n-1)2 = 4$

$\Rightarrow \quad a + 2n - 2 = 4$

$\Rightarrow \quad a = 4 - 2n + 2$

$\Rightarrow \quad a = 6 - 2n$ ...(1)

Also $\quad S_n = -14$

$\Rightarrow \quad \frac{n}{2}(a + l) = -14$

$\Rightarrow \quad \frac{n}{2}(a + 4) = -14$

$\Rightarrow \quad n(a + 4) = -28$ ...(2)

Substituting the value of a from (1) into (2),

$n[6 - 2n + 4] = -28$

$\Rightarrow \quad n[10 - 2n] = -28$

$\Rightarrow \quad 2n[5 - n] = -28$

$\Rightarrow \quad n(5 - n) = -14$ [Dividing throughout by 2]

$\Rightarrow \quad 5n - n^2 + 14 = 0$

$\Rightarrow \quad n^2 - 5n - 14 = 0$

$\Rightarrow \quad n^2 - 7n + 2n - 14 = 0$

$\Rightarrow \quad n(n - 7) + 2(n - 7) = 0$

$\Rightarrow \quad (n - 7)(n + 2) = 0$

$\therefore$ Either $\quad n - 7 = 0 \Rightarrow n = 7$

or $\quad n + 2 = 0 \Rightarrow n = -2$

But n cannot be negative,

$\therefore \quad n = 7$

Now, from (1), we have

$a = 6 - 2 \times 7 \Rightarrow a = -8$

Thus, $\quad \boldsymbol{a = -8}$ **and** $\boldsymbol{n = 7}$

(*ix*) Here, $\quad a = 3, \; n = 8$ and $S_n = 192$

Let the common difference = d.

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad 192 = \frac{8}{2}[2(3) + (8 - 1)d]$

$\Rightarrow \quad 192 = 4[6 + 7d]$

$\Rightarrow \quad 192 = 24 + 28d$

$\Rightarrow \quad 28d = 192 - 24 = 168$

$\Rightarrow \quad d = \frac{168}{28} = 6$

Thus, $\quad \boldsymbol{d = 6.}$

(*x*) Here, $\quad l = 28$ and $S_9 = 144$

Let the first term be 'a'.

Then $S_n = \frac{n}{2}(a + l)$

$\Rightarrow \quad S_9 = \frac{9}{2}(a + 28)$

$\Rightarrow \quad 144 = \frac{9}{2}(a + 28)$

$\Rightarrow \quad a + 28 = 144 \times \frac{2}{9} = 16 \times 2 = 32$

$\Rightarrow \quad a = 32 - 28 = 4$

Thus, $\boldsymbol{a = 4}$.

Q. 4. *How many terms of the A.P.: 9, 17, 25, ... must be taken to give a sum of 636?*

Sol. Here, $a = 9$

$d = 17 - 9 = 8$

$S_n = 636$

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d] = 636$

$\therefore \quad \frac{n}{2}[(2 \times 9) + (n - 1) \times 8] = 636$

$\Rightarrow \quad n[18 + (n - 1) \times 8] = 1272$

$\Rightarrow \quad n(8n + 10) = 1272$

$\Rightarrow \quad 8n^2 + 10n - 1272 = 0$

$\Rightarrow \quad 4n^2 + 5n - 636 = 0$

$\Rightarrow \quad 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow \quad n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow \quad (n - 12)(4n + 53) = 0 \quad \Rightarrow \quad n = 12$ and $n = -\frac{53}{4}$

Rejecting $n = -\frac{53}{4}$, we have $n = 12$.

Q. 5. *The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.*

Sol. Here, $a = 5$

$l = 45 = T_n$

$S_n = 400$

$\because \quad T_n = a + (n - 1)d$

$\therefore \quad 45 = 5 + (n - 1)d$

$\Rightarrow \quad (n - 1)d = 45 - 5$

$\Rightarrow \quad (n - 1)d = 40$ ...(1)

Also $S_n = \frac{n}{2}(a + l)$

$\Rightarrow \quad 400 = \frac{n}{2}(5 + 45)$

$\Rightarrow \quad 400 \times 2 = n \times 50$

$\Rightarrow \quad n = \frac{400 \times 2}{50} = 16$

From (1), we get

$(16 - 1)\, d = 40$

$\Rightarrow \quad 15d = 40$

$\Rightarrow \quad d = \frac{40}{15} = \frac{8}{3}$

Q. 6. *The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?*

Sol. We have,

First term $a = 17$

Last term $l = 350 = T_n$

Common difference $d = 9$

Let the number of terms be 'n'

$\because \quad T_n = a + (n - 1)\, d$

$\therefore \quad 350 = 17 + (n - 1) \times 9$

$\Rightarrow \quad (n - 1) \times 9 = 350 - 17 = 333$

$\Rightarrow \quad n - 1 = \frac{333}{9} = 37$

$\Rightarrow \quad n = 37 + 1 = 38$

Since, $\quad S_n = \frac{n}{2}\,(a + l)$

$\therefore \quad S_{38} = \frac{38}{2}\,(17 + 350)$

$= 19\,(367) = 6973$

Thus, $\quad \mathbf{n = 38}$ and $\mathbf{S_n = 6973}$

Q. 7. *Find the sum of first 22 terms of an A.P. in which d = 7 and 22nd term is 149.*

Sol. Here, $\quad n = 22, \quad T_{22} = 149 = l$

$d = 7$

Let the first term of the *A.P.* be 'a'.

$\therefore \quad T_n = a + (n - 1)\, d$

$\Rightarrow \quad T_n = a + (22 - 1) \times 7$

$\Rightarrow \quad a + 21 \times 7 = 149$

$\Rightarrow \quad a + 147 = 149$

$\Rightarrow \quad a = 149 - 147 = 2$

Now, $\quad S_{22} = \frac{n}{2}\,[a + l]$

$\Rightarrow \quad S_{22} = \frac{22}{2}\,[2 + 149]$

$= 11\,[151] = 1661$

Thus $\quad \mathbf{S_{22} = 1661}$

Q. 8. *Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.*

Sol. Here, $n = 51, \quad T_2 = 14 \quad \text{and} \quad T_3 = 18$

Let the first term of the A.P. be 'a' and the common difference is d.

$\therefore$ We have:

$T_2 = a + d \quad \Rightarrow \quad a + d = 14$ ...(1)

$T_3 = a + 2d \quad \Rightarrow \quad a + 2d = 18$ ...(2)

Subtracting (1) from 2, we get

$a + 2d - a - d = 18 - 14$

$\Rightarrow \quad d = 14$

From (1), we get

$a + d = 14 \quad \Rightarrow \quad a + 4 = 14$

$\Rightarrow \quad a = 14 - 4 = 10$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{51} = \frac{51}{2}[(2 \times 10) + (51 - 1) \times 4]$

$= \frac{51}{2}[20 + 200]$

$= \frac{51}{2}[220]$

$= 51 \times 110 = 5610$

Thus, the sum of 51 terms is **5610.**

Q. 9. *If the sum of first 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of first n terms.*

Sol. Here, we have:

$S_7 = 49 \quad \text{and} \quad S_{17} = 289$

Let the first term of the A.P. be 'a' and 'd' be the common difference, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad S_7 = \frac{7}{2}[2a + (7-1)d] = 49$

$\Rightarrow \quad 7(2a + 6d) = 2 \times 49 = 98$

$\Rightarrow \quad 2a + 6d = \frac{98}{7} = 14$

$\Rightarrow \quad 2[a + 3d] = 14$

$\Rightarrow \quad a + 3d = \frac{14}{2} = 7$

$\Rightarrow \quad a + 3d = 7$ ...(1)

Also, $S_{17} = \frac{17}{2}[2a + (17-1)d] = 289$

$$\Rightarrow \quad \frac{17}{2}(2a + 16d) = 289$$

$$\Rightarrow \quad a + 8d = \frac{289}{17} = 17$$

$$\Rightarrow \quad a + 8d = 17 \qquad \text{...(2)}$$

Subtracting (1) from (2), we have:

$$a + 8d - a - 3d = 17 - 7$$

$$\Rightarrow \quad 5d = 10$$

$$\Rightarrow \quad d = \frac{10}{5} = 2$$

Now, from (1), we have

$$a + 3(2) = 7$$

$$\Rightarrow \quad a = 7 - 6 = 1$$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 2]$$

$$= \frac{n}{2}[2 + 2n - 2]$$

$$= \frac{n}{2}[2n]$$

$$= n \times n = n^2$$

Thus, the required sum of n terms = $\boldsymbol{n^2}$.

Q. 10. *Show that $a_1, a_2, ..., a_n, ...$ form an A.P. where a_n is defined as below:*

(i) $a_n = 3 + 4n$ *(ii) $a_n = 9 - 5n$*

Also find the sum of the first 15 terms in each case.

Sol. (*i*) Here, $a_n = 3 + 4n$

Putting $n = 1, 2, 3, 4, ... n,$ we get:

$$a_1 = 3 + 4(1) = 7$$
$$a_2 = 3 + 4(2) = 11$$
$$a_3 = 3 + 4(3) = 15$$
$$a_4 = 3 + 4(4) = 19$$

....

$$a_n = 3 + 4n$$

∴ The A.P. in which $a = 7$ and $d = 11 - 7 = 4$ is:

7, 11, 15, 19, ..., $(3 + 4n)$.

Now $S_{15} = \frac{15}{2}[(2 \times 7) + (15 - 1) \times 4]$

$$= \frac{15}{2}[14 + (14 \times 4)]$$

$$= \frac{15}{2} [14 + 56]$$

$$= \frac{15}{2}[70]$$

$$= 15 \times 35 = \mathbf{525}$$

(*ii*) Here, $a_n = 9 - 5n$

Putting $n = 1, 2, 3, 4, \ldots, n$, we get

$$a_1 = 9 - 5(1) = 4$$

$$a_2 = 9 - 5(2) = -1$$

$$a_3 = 9 - 5(3) = -6$$

$$a_4 = 9 - 5(4) = -11$$

....

∴ The A.P. is:

4, – 1, – 6, – 11, ... 9 – 5 (n) [*having first term as 4 and* $d = -1 - 4 = -5$]

$$\therefore \quad S_{15} = \frac{15}{2} [(2 \times 4) + (15 - 1) \times (-5)]$$

$$= \frac{15}{2} [8 + 14 \times (-5)]$$

$$= \frac{15}{2} [8 - 70]$$

$$= \frac{15}{2} \times (-62)$$

$$= 15 \times (-31) = \mathbf{-465.}$$

Q. 11. *If the sum of the first n terms of an A.P. is* $4n - n^2$*, what is the first term (that is* S_1*)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.*

Sol. We have:

$$S_n = 4n - n^2$$

$$\therefore \quad S_1 = 4(1) - (1)^2$$

$$= 4 - 1 = 3 \Rightarrow \text{First term} = \mathbf{3}$$

$$S_2 = 4(2) - (2)^2$$

$$= 8 - 4 = 4 \Rightarrow \text{Sum of first two terms} = 4$$

∴ Second term $(S_2 - S_1) = 4 - 3 = \mathbf{1}$

$$S_3 = 4(3) - (3)^2$$

$$= 12 - 9 = \mathbf{3} \Rightarrow \text{Sum of first 3 terms} = 3$$

∴ Third term $(S_3 - S_2) = 3 - 4 = \mathbf{-1}$

$$S_9 = 4(9) - (9)^2$$

$$= 36 - 81 = -45$$

$$S_{10} = 4(10) - (10)^2$$

$$= 40 - 100 = \mathbf{-60}$$

∴ Tenth term $= S_{10} - S_9 = [-60] - [-45] = \mathbf{-15}$

Now, $S_n = 4(n) - (n)^2 = \mathbf{4n - n^2}$

Also $S_{n-1} = 4(n-1) - (n-1)^2$

$= 4n - 4 - [n^2 - 2n + 1]$

$= 4n - 4 - n^2 + 2n - 1$

$= 6n - n^2 - 5$

$\therefore$ nth term $= S_n - S_{n-1}$

$= [4n - n^2] - [6n - n^2 - 5]$

$= 4n - n^2 - 6n + n^2 + 5 = \mathbf{5 - 2n}$

Thus, $S_1 = 3$ and $a_1 = 3$

$S_2 = 4$ and $a_2 = 1$

$S_3 = 3$ and $a_3 = -1$

$a_{10} = -15$ and $a_n = 5 - 2n$

Q. 12. *Find the sum of the first 40 positive integers divisible by 6.*

Sol. $\because$ The first 40 positive integers divisible by 6 are:

6, 12, 18, ..., (6 × 40).

And, these numbers are in A.P. such that

$a = 6$

$d = 12 - 6 = 6$ and $a_n = 6 \times 40 = 240 = l$

$$\therefore \quad S_{40} = \frac{40}{2}[(2 \times 6) + (40 - 1) \times 6]$$

$= 20\,[12 + 39 \times 6]$

$= 20\,[12 + 234]$

$= 20 \times 246 = \mathbf{4920}$

OR

$$S_n = \frac{n}{2}[a + l]$$

$$S_{40} = \frac{40}{2}[6 + 240]$$

$= 20 \times 246 = \mathbf{4920}$

Thus, the sum of first 40 multiples of 6 is **4920.**

Q. 13. *Find the sum of the first 15 multiples of 8.*

Sol. The first 15 multiples of 8 are:

8, (8 × 2), (8 × 3), (8 × 4), ..., (8 × 15)

or 8, 16, 24, 32, ..., 120.

These numbers are in A.P., where

$a = 8$ and $l = 120$

$$\therefore \quad S_{15} = \frac{15}{2}[a + l]$$

$$= \frac{15}{2}[8 + 120]$$

$$= \frac{15}{2} \times 128$$

$= 15 \times 64 = 960$

Thus, the sum of first positive 15 multiples of 8 is **960.**

Q. 14. *Find the sum of the odd numbers between 0 and 50.*

Sol. Odd numbers between 0 and 50 are:

1, 3, 5, 7, ..., 49

These numbers are in A.P. such that

$a = 1$ and $l = 49$

Here, $d = 3 - 1 = 2$

$\therefore \quad T_n = a + (n-1)d$

$\Rightarrow \quad 49 = 1 + (n-1)2$

$\Rightarrow \quad 49 - 1 = (n-1)2$

$\Rightarrow \quad (n-1) = \frac{48}{2} = 24$

$\therefore \quad n = 24 + 1 = 25$

Now, $S_{25} = \frac{25}{2}[1 + 49]$

$= \frac{25}{2}[50]$

$= 25 \times 25 = 625$

Thus, the sum of odd numbers between 0 and 50 is **625.**

Q. 15. *A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?* (CBSE 2012)

Sol. Here, penalty for delay on

1st day = ₹ 200

2nd day = ₹ 250

3rd day = ₹ 300

....

....

Now, 200, 250, 300, ... are in A.P. such that

$a = 200, \quad d = 250 - 200 = 50$

$\therefore S_{30}$ is given by

$S_{30} = \frac{30}{2}[2(200) + (30-1) \times 50]$ $\left[\text{using } S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 15[400 + 29 \times 50]$

$= 15[400 + 1450]$

$= 15 \times 1850 = 27,750$

Thus, penalty for the delay for 30 days is ₹ **27,750.**

Q. 16. *A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performace. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.* (CBSE 2012)

Sol. Sum of all the prizes = ₹ 700

Let the first prize = a

$\therefore$ 2nd prize = $(a - 20)$

3rd prize = $(a - 40)$

4th prize = $(a - 60)$

....

Thus, we have, first term = a

Common difference = – 20

Number of prizes, $n = 7$

Sum of 7 terms $S_n = 700$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 700 = \frac{7}{2}[2(a) + (7-1) \times (-20)]$

$\Rightarrow \quad 700 = \frac{7}{2}[2a + (6 \times -20)]$

$\Rightarrow \quad 700 \times \frac{2}{7} = 2a - 120$

$\Rightarrow \quad 200 = 2a - 120$

$\Rightarrow \quad 2a = 200 + 120 = 320$

$\Rightarrow \quad a = \frac{320}{2} = 160$

Thus, the values of the seven prizes are:

₹ 160, ₹ (160 – 20), ₹ (160 – 40), ₹ (160 – 60), ₹ (160 – 80), ₹ (160 – 100) and ₹ (160 – 120)

$\Rightarrow$ **₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60** and **₹ 40.**

Q. 17. *In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?* (CBSE 2012)

Sol. Number of classes = 12

$\because$ Each class has 3 sections.

$\therefore$ Number of plants planted by class I = 1 × 3 = 3

Number of plants planted by class II = 2 × 3 = 6

Number of plants planted by class III = 3 × 3 = 9

Number of plants planted by class IV = 4 × 3 = 12

....

Number of plants planted by class XII = 12 × 3 = 36

The numbers 3, 6, 9, 12, ..., 36 are in A.P.

Here, $a = 3$ and $d = 6 - 3 = 3$

$\because$ Number of classes = 12

i.e., $n = 12$

$\therefore$ Sum of the n terms of the above A.P., is given by

$$S_{12} = \frac{12}{2}\ [2\ (3) + (12 - 1)\ 3] \qquad \left[\text{using } S_n = \frac{n}{2}\ [2a + (n-1)d]\right]$$

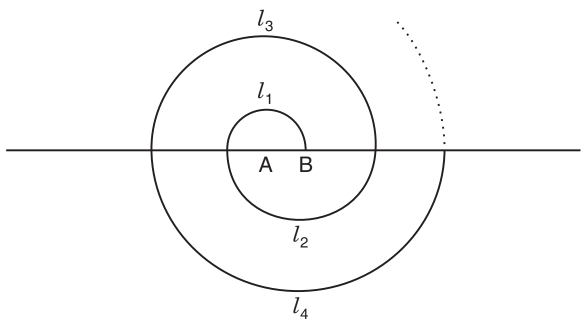
$$= 6\ [6 + 11 \times 3]$$

$$= 6\ [6 + 33]$$

$$= 6 \times 39 = 234$$

Thus, the total number of trees = **234.**

Q. 18. *A spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semi-circles? (Take* $\pi = \frac{22}{7}$*)*



*[**Hint:** Length of successive semi-circles is* l_1, l_2, l_3, l_4, *... with centres at A, B, A, B, ..., respectively.]*

Sol. $\because$ Length of a semi-circle = semi-circumference

$$= \frac{1}{2}\ (2\pi\ r)$$

$$= \pi\ r$$

$\therefore$ $l_1 = \pi\ r_1 = 0.5\ \pi$ cm $= 1 \times 0.5\ \pi$ cm

$l_2 = \pi\ r_2 = 1.0\ \pi$ cm $= 2 \times 0.5\ \pi$ cm

$l_3 = \pi\ r_3 = 1.5\ \pi$ cm $= 3 \times 0.5\ \pi$ cm

$l_4 = \pi\ r_4 = 2.0\ \pi$ cm $= 4 \times 0.5\ \pi$ cm

....

$l_{13} = \pi\ r_{13}$ cm $= 6.5\ \pi$ cm $= 13 \times 0.5\ \pi$ cm

Now, length of the spiral

$$= l_1 + l_2 + l_3 + l_4 + ... + l_{13}$$

$$= 0.5\ \pi\ [1 + 2 + 3 + 4 + ... + 13] \text{ cm} \qquad ...(1)$$

$\because$ 1, 2, 3, 4, ..., 13 are in A.P. such that

$a = 1$ and $l = 13$

$$\therefore \quad S_{13} = \frac{13}{2}[1 + 13] \qquad \left[\textit{using } S_n = \frac{n}{2}\ (a + l)\right]$$

$$= \frac{13}{2} \times 14$$

$$= 13 \times 7 = 91$$

∴ From (1), we have:

Total length of the spiral

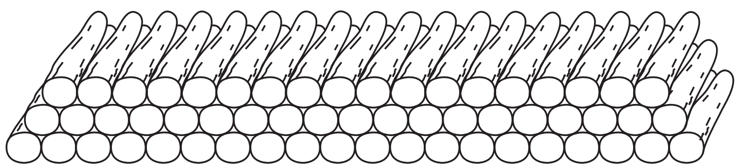
$$= 1.5\,\pi\,[91] \text{ cm}$$

$$= \frac{5}{10} \times \frac{22}{7} \times 91 \text{ cm} \qquad \left[\because \pi = \frac{22}{7}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times 91 \text{ cm}$$

$= 11 \times 13$ cm = **143 cm.**

Q. 19. *200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?*



Sol. We have:

The number of logs:

1st row = 20

2nd row = 19

3rd row = 18

obviously, the numbers

20, 19, 18, ..., are in A.P. such that

$a = 20$

$d = 19 - 20 = -1$

Let the numbers of rows be n.

$\therefore \quad S_n = 200$

Now, using, $S_n = \frac{n}{2}\,[2a + (n-1)\,d]$, we get

$$S_n = \frac{n}{2}\,[2\,(20) + (n-1) \times (-1)]$$

$$\Rightarrow \quad 200 = \frac{n}{2}\,[40 - (n-1)]$$

$$\Rightarrow \quad 2 \times 200 = n \times 40 - n\,(n-1)$$

$$\Rightarrow \quad 400 = 40n - n^2 + n$$

$$\Rightarrow \quad n^2 - 41n + 400 = 0$$

$$\Rightarrow \quad n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow \quad n\,(n-16) - 25\,(n-16) = 0$$

$$\Rightarrow \quad (n-16)\,(n-25) = 0$$

Either

$\Rightarrow n - 16 = 0 \Rightarrow n = 16$

or $n - 25 = 0 \Rightarrow n = 25$

$T_n = 0 \Rightarrow a + (n-1)d = 0 \Rightarrow 20 + (n-1) \times (-1) = 0$

$\Rightarrow n - 1 = 20 \Rightarrow n = 21$

i.e., 21st term becomes 0

$\therefore$ $n = 25$ is not required.

Thus, $n = 16$

$\therefore$ Number of rows = **16**

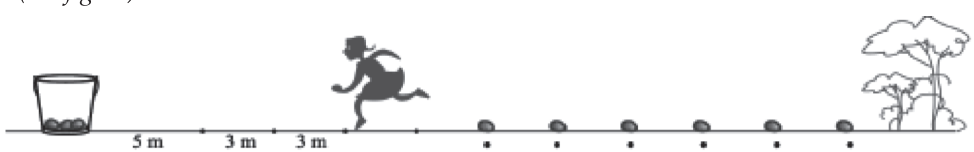
Now, $T_{16} = a + (16-1)d$

$= 20 + 15 \times (-1)$

$= 20 - 15 = 5$

$\therefore$ Number of logs in the 16th (top) row is **15.**

Q. 20. *In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see figure).*



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

*[**Hint:** To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]*

Sol. Here, number of potatoes = 10

The up-down distance of the bucket:

From the 1st potato = [5 m] × 2 = 10 m

From the 2nd potato = [(5 + 3) m] × 2 = 16 m

From the 3rd potato = [(5 + 3 + 3) m] × 2 = 22 m

From the 4th potato = [(5 + 3 + 3 + 3) m] × 2 = 28 m

....

$\because$ 10, 16, 22, 28, ... are in A.P. such that

$a = 10$ and $d = 16 - 10 = 6$

$\therefore$ Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we have:

$S_{10} = \frac{10}{2}[2(10) + (10-1) \times 6]$

$= 5[20 + 9 \times 6]$

$= 5\ [20 + 54]$
$= 5\ [74]$
$= 5 \times 74 = 370$

Thus, the sum of above distances = 370 m.

$\Rightarrow$ The competitor has to run a total distance of **370 m.**

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 5.4

Q. 1. *Which term of the A.P.: 121, 117, 113, ..., is its first negative term?*
*[**Hint:** Find n for $a_n < 0$]* (CBSE 2012)

Sol. We have the A.P. having $a = 121$ and $d = 117 - 121 = -4$

$\therefore \quad a_n = a + (n - 1)\ d$
$= 121 + (n - 1) \times (-4)$
$= 121 - 4n + 4$
$= 125 - 4n$

For the first negative term, we have

$a_n < 0$
$\Rightarrow (125 - 4n) < 0$
$\Rightarrow 125 < 4n$
$\Rightarrow \dfrac{125}{4} < n$
$\Rightarrow 31\dfrac{1}{4} < n$
or $n > 31\dfrac{1}{4}$

Thus, the first negative term is 32nd term.

Q. 2. *The sum of the third and the seventh terms of an A.P. is 6 and their product is 8. Find the sum of first sixteen terms of the A.P..*

Sol. Here, $T_3 + T_7 = 6$ and $T_3 \times T_7 = 8$

Let the first term = a and the common difference = d

$\therefore \quad T_3 = a + 2d$ and $T_7 = a + 6d$
$\because \quad T_3 + T_7 = 6$
$\therefore \quad (a + 2d) + (a + 6d) = 6$
$\Rightarrow \quad 2a + 8d = 6$
$\Rightarrow \quad a + 4d = 3$ ...(1)

Again $T_3 \times T_7 = 8$
$\therefore \quad (a + 2d) \times (a + 6d) = 8$
$\Rightarrow \quad (a + 4d - 2d) \times (a + 4d + 2d) = 8$
$\Rightarrow \quad [(a + 4d) - 2d] \times [(a + 4d) + 2d] = 8$
$\Rightarrow \quad [(3) - 2d] \times [(3) + 2d] = 8$ [*From (1)*]
$\Rightarrow \quad 3^2 - (2d)^2 = 8$
$\Rightarrow \quad 9 - 4d^2 = 8$

$\Rightarrow \quad -4d^2 = 8 - 9 = -1$

$\Rightarrow \quad d^2 = \frac{-1}{-4} = \frac{1}{4}$

$\Rightarrow \quad d = \pm\frac{1}{2}$

When $d = \frac{1}{2}$.

From (1), we have:

$$a + 4\left(\frac{1}{2}\right) = 3$$

$\Rightarrow \quad a + 2 = 3$ or $a = 3 - 2 = 1$

Now, Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$S_{16} = \frac{16}{2}\left[2(1) + (16-1)\times\frac{1}{2}\right]$$

$$= 8\left[2 + \frac{15}{2}\right]$$

$$= 16 + 60 = 76$$

i.e., the sum of first 16 terms = **76**

When $d = -\frac{1}{2}$.

From (1), we have:

$$a + 4\left(-\frac{1}{2}\right) = 3$$

$\Rightarrow \quad a - 2 = 3 \Rightarrow a = 5$

Again, the sum of first sixteen terms

$$S_{16} = \frac{16}{2}\left[2(5) + (16-1)\times\left(-\frac{1}{2}\right)\right]$$

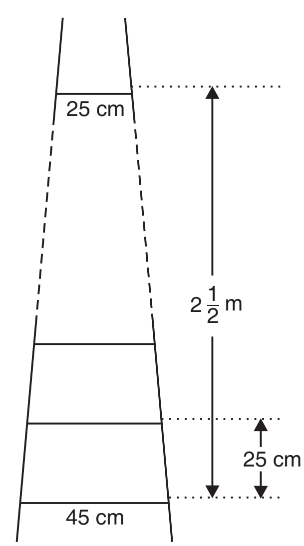
$$= 8\left[10 + \left(\frac{-15}{2}\right)\right]$$

$$= 80 - 60 = 20$$

i.e., the sum of first 16 terms = **20**

Q. 3. *A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?*

*[**Hint:** Number of rungs = $\frac{250}{25} + 1$]*



Sol. Total distance between bottom to top rungs

$$= 2\frac{1}{2}\text{ m}$$

$$= \frac{5}{2} \times 100 \text{ cm}$$

$= 250$ m

Distance between two consecutive rungs = 25 cm

$\therefore$ Number of rungs = $\frac{250}{25} + 1$ = 10 + 1 = 11

Length of the 1st rung (bottom rung) = 45 cm

Length of the 11th rung (top rung) = 25 cm

Let the length of each successive rung decrease by x cm

$\therefore$ Total length of the rungs

= 45 cm + (45 – x) cm + (45 – 2x) cm + ... + 25 cm

Here, the numbers 45, (45 – x), (45 – 2x), ..., 25 are in an A.P. such that

First term 'a' = 45

Last term 'l' = 25

Number of terms 'n' = 11

$\therefore$ Using, $S_n = \frac{n}{2}[a + l]$, we have

$$S_{11} = \frac{11}{2}[45 + 25]$$

$$\Rightarrow \quad S_{11} = \frac{11}{2} \times 70$$

$$\Rightarrow \quad S_{11} = 11 \times 35 = 385$$

$\therefore$ Total length of 11 rungs = 385 cm

i.e., Length of wood required for the rungs is **385 cm.**

Q. 4. *The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.*

*[**Hint:** $S_{x-1} = S_{49} - S_x$]*

Sol. We have, the following consecutive numbers on the houses of a row ;

1, 2, 3, 4, 5, ..., 49.

These numbers are in an A.P., such that

$a = 1$

$d = 2 - 1 = 1$

$n = 49$

Let one of the houses be numbered as x

$\therefore$ Number of houses preceding it = $x - 1$

Number of houses following it = $49 - x$

Now, the sum of the house-numbers preceding x is given by:

Using $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{x-1} = \frac{x-1}{2}[2(1) + (x - 1 - 1) \times 1]$$

$$= \frac{x-1}{2}[2 + x - 2]$$

$$= \frac{x-1}{2}[x]$$

$$= \frac{x(x-1)}{2}$$

$$= \frac{x^2}{2} - \frac{x}{2}$$

The houses beyond x are numbered as

$(x + 1), (x + 2), (x + 3), \ldots, 49$

∴ For these house numbers (which are in an A.P.),

First term $(a) = x + 1$

Last term $(l) = 49$

∴ Using $S_n = \frac{n}{2}[a + l]$, we have

$$S_{49-x} = \frac{49-x}{2}[(x + 1) + 49]$$

$$= \frac{49-x}{2}[x+50]$$

$$= \frac{49x}{2} - \frac{x^2}{2} + (49 \times 25) - 25x$$

$$= \left(\frac{49x}{2} - 25x\right) - \frac{x^2}{2} + (49 \times 25)$$

$$= \frac{-x}{2} - \frac{x^2}{2} + (49 \times 25)$$

According to the question,

[Sum of house numbers preceding x] = [Sum of house numbers following x]

i.e., $S_{x-1} = S_{49-x}$

$$\Rightarrow \frac{x^2}{2} - \frac{x}{2} = \frac{-x}{2} - \frac{x^2}{2} + (49 \times 25)$$

$$\Rightarrow \left(\frac{x^2}{2} + \frac{x^2}{2}\right) - \frac{x}{2} + \frac{x}{2} = (49 \times 25)$$

$$\Rightarrow \frac{2x^2}{2} = (49 \times 25)$$

$$\Rightarrow x^2 = (49 \times 25)$$

$$\Rightarrow x = \pm\sqrt{49 \times 25}$$

$$\Rightarrow x = \pm (7 \times 5) = \pm 35$$

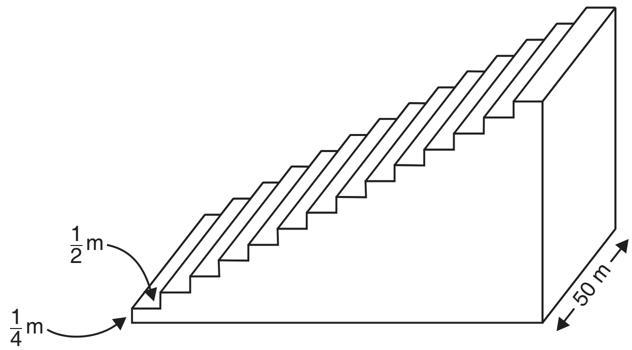
But x cannot be taken as –ve

∴ $x = \mathbf{35}$

Q. 5. *A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.*

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see Fig.). Calculate the total volume of concrete required to build the terrace.

*[**Hint:** Volume of concrete required to build the first step = $\frac{1}{4} \times \frac{1}{2} \times 50\ m^3$]*



Sol. For 1st step:

Length = 50 m, Breadth = $\frac{1}{2}$ m, Height = $\frac{1}{4}$ m

∴ Volume of concrete required to build the 1st step

= Volume of the cuboidal step

= Length × Breadth × height

$= 50 \times \frac{1}{2} \times \frac{1}{4}\ m^3$

$= \frac{25}{4} \times 1\ m^3$

For 2nd step:

Length = 50 m, Breadth = $\frac{1}{2}$ m, Height = $\left(\frac{1}{4} + \frac{1}{4}\right)$ m $= 2 \times \frac{1}{4}$ m

∴ Volume of concrete required to build the 2nd step

$= 50 \times \frac{1}{2} \times \frac{1}{4} \times 2\ m^3$

$= \frac{25}{4} \times 2\ m^3$

For 3rd step:

Length = 50 m, Breadth = $\frac{1}{2}$ m, Height = $\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$ m $= 3 \times \frac{1}{4}$ m

∴ Volume of concrete required to build the 3rd step

$= 50 \times \frac{1}{2} \times \frac{1}{4} \times 3\ m^3$

$= \frac{25}{4} \times 3\ m^3$

....

Thus, the volumes (in m^3) of concrete required to build the various steps are:

$$\left(\frac{25}{4}\times 1\right), \left(\frac{25}{4}\times 2\right), \left(\frac{25}{4}\times 3\right), ...$$

obviously, these numbers form an A.P. such that

$$a = \frac{25}{4}$$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

Here, total number of steps $n = 15$

Total volume of concrete required to build 15 steps is given by the sum of their individual volumes.

$\therefore$ Using, $S_n = \frac{n}{2}[2(a) + (n-1)d]$, we have:

$$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\times\frac{25}{4}\right] \text{m}^3$$

$$= \frac{15}{2}\left[\frac{25}{2} + 14\times\frac{25}{4}\right] \text{m}^3$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right] \text{m}^3$$

$$= \frac{15}{2}\times\frac{200}{2} \text{m}^3$$

$$= 15 \times 50 \text{ m}^3 = 750 \text{ m}^3$$

Thus, the required volume of concrete is **750 m^3.**

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *If the numbers $x - 2$, $4x - 1$ and $5x + 2$ are in A.P. Find the value of x.*

[NCERT Exemplar Problem, (CBSE Sample Paper 2011)]

Sol. $\because$ $x - 2$, $4x - 1$ and $5x + 2$ are in A.P.

$\therefore$ $(4x - 1) - (x - 2) = (5x + 2) - (4x - 1)$

$\Rightarrow$ $3x + 1 = x + 3$

$\Rightarrow$ $2x = 2 \Rightarrow x = 1$

Q. 2. *Which term of the A.P. 4, 9, 14, ... is 109?*

Sol. Let 109 is the nth term,

$\therefore$ Using $T_n = a + (n - 1)d$, we have:

$109 = 4 + (n - 1) 5$ [$\because$ *$a = 4$ and $d = 9 - 4 = 5$*]

$$\Rightarrow \quad n - 1 = \frac{109-4}{5} = \frac{105}{5} = 21$$

$$\Rightarrow \quad n = 21 + 1 = 22$$

Thus, the **22nd** term is 109.

Q. 3. *If a, (a – 2) and 3a are in A.P. then what is the value of a?*

Sol. $\because$ a, $(a - 2)$ and $3a$ are in A.P.

$\therefore \quad (a - 2) - a = 3a - (a - 2)$

$\Rightarrow \quad a - 2 - a = 3a - a + 2$

$\Rightarrow \quad -2 = 2a + 2$

$\Rightarrow \quad 2a = -2 - 2 = -4$

$$\Rightarrow \quad a = \frac{-4}{2} = -2$$

Thus, the required value of a is **– 2.**

Q. 4. *How many terms are there in the A.P.?*

7, 10, 13, ..., 151

Sol. Here, $a = 7, \quad d = 10 - 7 = 3$

Let there are n-terms.

$\therefore \quad T_n = a + (n - 1)\,d$

$\Rightarrow \quad T_{51} = 7 + (n - 1) \times 3$

$$\Rightarrow \quad \frac{151-7}{3} = n - 1$$

$$\Rightarrow \quad \frac{144}{3} = n - 1 \quad \Rightarrow \quad n = 48 + 1 = 49$$

i.e., $\quad$ **$n = 49$**

Q. 5. *Which term of the A.P. 72, 63, 54, ... is 0?*

Sol. Here, $a = 72$

$d = 63 - 72 = -9$

Let nth term of this A.P. be 0

$\therefore \quad T_n = a + (n - 1)\,d$

$\Rightarrow \quad 72 + (n - 1) \times (-9) = 0$

$$\Rightarrow \quad (n - 1) = \frac{-72}{-9} = 8$$

$\Rightarrow \quad n = 8 + 1 = 9$

Thus the 9th term of the A.P. is 0.

Q. 6. *The first term of an A.P. is 6 and its common difference is – 2. Find its 18th term.*

Sol. Using $T_n = a + (n - 1)\,d$, we have:

$T_{18} = 6 + (18 - 1) \times (-2)$

$= 6 + 17 \times (-2)$

$= 6 - 34 = -28$

Thus, the 18th term is **– 28.**

Q. 7. *The 4th term of an A.P. is 14 and its 12th term 70. What is its first term?*

Sol. Let the first term = a

If 'd' is the common difference,

Then $\quad T_4 = a + 3d = 14 \quad$ ...(1)

And $\quad T_{12} = a + 11d = 70 \quad$ ...(2)

Subtracting (1) from (2),

$$a + 11d - a - 3d = 70 - 14$$

$$\Rightarrow \quad 8d = 56 \quad \Rightarrow \quad d = \frac{56}{8} = 7$$

$\therefore$ From (1), $a + 3(7) = 14$

$$\Rightarrow \quad a + 21 = 14$$

$$\Rightarrow \quad a = 14 - 21 = (-7)$$

Thus, the first term is **– 7.**

Q. 8. *Which term of A.P. 5, 2, – 1, – 4 ..... is – 40?*

Sol. Here, $a = 5$

$$d = 2 - 5 = -3$$

Let nth term be – 40

$$\therefore \quad T_n = a + (n - 1)\, d$$

$$\Rightarrow \quad -40 = 5 + (n - 1) \times (-3)$$

$$\Rightarrow \quad n - 1 = \frac{-40 - 5}{-3} = \frac{-45}{-3} = 15$$

$$\Rightarrow \quad n = 15 + 1 = 16$$

i.e., The **16th** term of the A.P. is – 40.

Q. 9. *What is the sum of all the natural numbers from 1 to 100?*

Sol. We have:

1, 2, 3, 4, ..., 100 are in an A.P. such that

$$a = 1 \text{ and } l = 100$$

$$\therefore \quad S_n = \frac{n}{2}[a + l]$$

$$\Rightarrow \quad S_{100} = \frac{100}{2}[1 + 100] = 50 \times 101 = \mathbf{5050.}$$

Q. 10. *For an A.P., the 8th term is 17 and the 14th term is 29. Find its common difference.*

Sol. Let the common difference = d and first term = a

$$\therefore \quad T_8 = a + 7d = 17 \quad ...(1)$$

$$T_{14} = a + 13d = 29 \quad ...(2)$$

Subtracting (1) from (2), we have:

$$a + 13d - a - 7d = 29 - 17$$

$$\Rightarrow \quad 6d = 12$$

$$\Rightarrow \quad d = \frac{12}{6} = 2$$

$\therefore$ The required common difference = **2.**

Q. 11. *If the first and last terms of an A.P. are 10 and – 10. How many terms are there? Given that $d = -1$.*

Sol. Let the required number of terms is n and

1st term $a = 10$

nth term $T_n = -10$

Let common difference be d then using,

$$T_n = a + (n - 1)\, d, \text{ we have:}$$

$$-10 = 10 + (n-1) \times (-1)$$
$$\Rightarrow \quad -10 = 10 - n + 1$$
$$\Rightarrow \quad -n + 1 = -10 - 10 = -20$$
$$\Rightarrow \quad -n = -20 - 1 = -21$$
$$\Rightarrow \quad n = 21$$

Q. 12. *The nth term of an A.P. is (3n – 2) find its first term.*

Sol. $\because \quad T_n = 3n - 2$

$\therefore \quad T_1 = 3(1) - 2 = 3 - 2 = 1$

$\Rightarrow$ First term = 1

Q. 13. *The nth term of an A.P. is (2n – 3) find the common difference.*

Sol. Here, $T_n = 2n - 3$

$\therefore \quad T_1 = 2(1) - 3 = -1$

$T_2 = 2(2) - 3 = 1$

$\therefore \quad d = T_2 - T_1 = 1 - (-1) = 2$

Thus the common difference is **2.**

Q. 14. *If the nth term of an A.P. is (7n – 5). Find its 100th term.*

Sol. Here, $T_n = 7n - 5$

$\therefore \quad T_1 = 7(1) - 5 = 2$

$T_2 = 7(2) - 5 = 9$

$\therefore \quad a = 2$

and $\quad d = T_2 - T_1$

$= 9 - 2 = 7$

Now $T_{100} = 2 + (100 - 1)\,7$ $\qquad$ [*using* $T_n = a + (n-1)\,d$]

$= 2 + 99 \times 7$

$= 2 + 693 =$ **695.**

Q. 15. *Find the sum of first 12 terms of the A.P. 5, 8, 11, 14, ...*

Sol. Here, $a = 5$

$d = 8 - 5 = 3$

$n = 12$

Using $\quad S_n = \dfrac{n}{2}[2(a) + (n-1)\,d]$

we have: $\quad S_{12} = \dfrac{12}{2}[2(5) + (12-1) \times 3]$

$= 6\,[10 + 33]$

$= 6 \times 43 = 258$

Q. 16. *Write the common difference of an A.P. whose nth term is 3n + 5.* (AI CBSE 2009 C)

Sol. $T_n = 3n + 5$

$\therefore \quad T_1 = 3(1) + 5 = 8$

$T_2 = 3(2) + 5 = 11$

$\Rightarrow \quad d = T_2 - T_1$

$= 11 - 8 = 3$

Thus, the common difference = **3.**

Q. 17. *Write the value of x for which x + 2, 2x, 2x + 3 are three consecutive terms of an A.P.* (CBSE 2009 C)

Sol. Here, $T_1 = x + 2$

$T_2 = 2x$

$T_3 = 2x + 3$

For an A.P., we have:

$\therefore \quad 2x - (x + 2) = 2x + 3 - 2x$

$\Rightarrow \quad 2x - x - 2 = 2x + 3 - 2x$

$\Rightarrow \quad x - 2 = 3$

$\Rightarrow \quad x = 3 + 2 = 5$

Thus, $\boldsymbol{x = 5}$

Q. 18. *What is the common difference of an A.P. whose nth term is 3 + 5n?* (CBSE 2009 C)

Sol. $\because \quad T_n = 3 + 5n$

$\therefore \quad T_1 = 3 + 5\,(1) = 8$

And $\quad T_2 = 3 + 5\,(2) = 13$

$\because \quad d = T_2 - T_1$

$\therefore \quad d = 13 - 8 = 5$

Thus, common difference = **5.**

Q. 19. *For what value of k, are the numbers x, (2x + k) and (3x + 6) three consecutive terms of an A.P.?* (AI CBSE 2009)

Sol. Here, $T_1 = x$, $T_2 = (2x + k)$ and $T_3 = (3x + 6)$

For an A.P., we have

$T_2 - T_1 = T_3 - T_2$

i.e., $\quad 2x + k - x = 3x + 6 - (2x + k)$

$\Rightarrow \quad x + k = 3x + 6 - 2x - k$

$\Rightarrow \quad x + k = x + 6 - k$

$\Rightarrow \quad k + k = x + 6 - x$

$\Rightarrow \quad 2k = 6$

$\Rightarrow \quad k = \frac{6}{2} = \mathbf{3}$

Q. 20. *If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a?* (AI CBSE 2009)

Sol. Here, $T_1 = \frac{4}{5}$

$T_2 = a$

$T_3 = 2$

$\because$ For an A.P.,

$T_2 - T_1 = T_3 - T_2$

$\therefore \quad a - \frac{4}{5} = 2 - a$

$\Rightarrow \quad a + a = 2 + \frac{4}{5}$

$\Rightarrow \quad 2a = \frac{14}{5}$

$$\Rightarrow \quad a = \frac{14}{5} \times \frac{1}{2} = \frac{7}{5}$$

Thus, $\boldsymbol{a = \frac{7}{5}}$

Q. 21. *For what value of p are 2p – 1, 7 and 3p three consecutive terms of an A.P.?* (CBSE 2009)

Sol. Here,
$$T_1 = 2p - 1$$
$$T_2 = 7$$
$$T_3 = 3p$$

$\because$ For an A.P., we have:

$$T_2 - T_1 = T_3 - T_2$$
$$\Rightarrow \quad 7 - (2p - 1) = 3p - 7$$
$$\Rightarrow \quad 7 - 2p + 1 = 3p - 7$$
$$\Rightarrow \quad -2p - 3p = -7 - 1 - 7$$
$$\Rightarrow \quad -5p = -15$$
$$\Rightarrow \quad p = \frac{-15}{-5} = 3$$

Thus, $\boldsymbol{p = 3}$

Q. 22. *For what value of p are 2p + 1, 13 and 5p – 3 three consecutive terms of an A.P.?* (CBSE 2009)

Sol. Here,
$$T_1 = 2p + 1$$
$$T_2 = 13$$
$$T_3 = 5p - 3$$

For an A.P., we have:

$$T_2 - T_1 = T_3 - T_2$$
$$\Rightarrow \quad 13 - (2p + 1) = 5p - 3 - 13$$
$$\Rightarrow \quad 13 - 2p - 1 = 5p - 16$$
$$\Rightarrow \quad -2p + 12 = 5p - 16$$
$$\Rightarrow \quad -2p - 5p = -16 - 12 = -28$$
$$\Rightarrow \quad -7p = -28$$
$$\Rightarrow \quad p = \frac{-28}{-7} = \frac{28}{7} = 4$$
$$\therefore \quad \boldsymbol{p = 4}$$

Q. 23. *The nth term of an A.P. is 7 – 4n. Find its common difference.* (CBSE 2008)

Sol.
$$\because \quad T_n = 7 - 4n$$
$$\therefore \quad T_1 = 7 - 4(1) = 3$$
$$T_2 = 7 - 4(2) = -1$$
$$\therefore \quad d = T_2 - T_1$$
$$= (-1) - 3 = -4$$

Thus, common difference = **– 4**

Q. 24. *The nth term of an A.P. is 6n + 2. Find the common difference.* (CBSE 2008)

Sol. Here,
$$T_n = 6n + 2$$
$$\therefore \quad T_1 = 6(1) + 2 = 8$$
$$T_2 = 6(2) + 2 = 14$$
$$\Rightarrow \quad d = T_2 - T_1 = 14 - 8 = 6$$

$\therefore$ Common difference = **6.**

Q. 25. *Write the next term of the A.P.* $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$ (AI CBSE 2008)

Sol. Here, $T_1 = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

$T_2 = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

$T_3 = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$\therefore \quad a = 2\sqrt{2}$

Now, $d = T_2 - T_1$

$= 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}(3-2) = \sqrt{2}$

$\therefore \quad T_4 = a + 3d$

$= 2\sqrt{2} + 3\left(\sqrt{2}\right)$

$= 2\sqrt{2} + 3\sqrt{2}$

$= \sqrt{2}(2+3) = 5\sqrt{2}$ or $\sqrt{50}$

Thus, the next term of the A.P. is $\mathbf{5\sqrt{2}}$ or $\mathbf{\sqrt{50}}$.

Q. 26. *The first term of an A.P. is p and its common difference is q. Find the 10th term.*
(AI CBSE 2008)

Sol. Here, $a = p$ and $d = q$

$\because \quad T_n = a + (n-1)\, d$

$\therefore \quad T_{10} = p + (10-1)\, q$

$= p + 9q$

Thus, the 10th term is $\boldsymbol{p + 9q}$.

Q. 27. *Find the next term of the A.P.* $\sqrt{2}, \sqrt{8}, \sqrt{18}$ ..... (CBSE 2008 C)

Sol. Here, $T_1 = \sqrt{2} \Rightarrow a = \sqrt{2}$

$T_2 = \sqrt{8} = 2\sqrt{2}$

$T_3 = \sqrt{18} = 3\sqrt{2}$

Now, $d = T_2 - T_1$

$= 2\sqrt{2} - \sqrt{2}$

$= \sqrt{2}$

Now, using $T_n = a + (n-1) \times d$, we have

$T_4 = a + 3d$

$= \sqrt{2} + 3\left(\sqrt{2}\right)$

$= \sqrt{2}\left[1+3\right] = 4\sqrt{2}$

$= \sqrt{16 \times 2} = \sqrt{32}$

Thus, the next term = $\mathbf{\sqrt{32}}$.

Q. 28. *Which term of the A.P.:*

21, 18, 15, ... is zero? (CBSE 2008 C)

Sol. Here, $a = 21$

$d = 18 - 21 = -3$

Since $T_n = a + (n-1)\,d$

$\Rightarrow \quad 0 = 21 + (n-1) \times (-3)$

$\Rightarrow \quad -3\,(n-1) = -21$

$\Rightarrow \quad (n-1) = \frac{-21}{-3} = 7$

$\Rightarrow \quad n = 7 + 1 = 8$

Thus, the **8**th **term** of this A.P. will be 0.

Q. 29. *Which term of the A.P.:*

14, 11, 8, ... is – 1? (AI CBSE 2008 C)

Sol. Here, $a = 14$

$d = 11 - 14 = -3$

Let the nth term be (-1)

$\therefore$ Using $T_n = a + (n-1)\,d$, we get

$-1 = 11 + (n-1) \times (-3)$

$\Rightarrow \quad -1 - 14 = -3\,(n-1)$

$\Rightarrow \quad -15 = -3\,(n-1)$

$\therefore \quad n - 1 = \frac{-15}{-3} = 5$

$\Rightarrow \quad n = 5 + 1 = 6$

Thus, –1 is the 6th term of the A.P.

Q. 30. *The value of the middlemost term (s) of the AP : –11, –7, –3, ...49.*

[NCERT Exemplar]

Sol. $\because \quad a = -11, \; a_n = 49$ and $d = (-7) - (-11) = 4$

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 49 = -11 + (n-1) \times 4 \quad \Rightarrow \quad n = 16$

Since, n is an even number

$\therefore$ There will be two middle terms, which are:

$$\frac{16}{2}\text{th} \quad \text{and} \quad \left(\frac{16}{2}+1\right)\text{th}$$

or 8th and 9th

Now, $a_8 = a + (8-1)d$

$= -11 + 7 \times 4 = 17$

$a_9 = a + (9-1)d$

$= -11 + 8 \times 4 = 21$

Thus, the values of the two middlemost terms are : **17** and **21**.

II. SHORT ANSWER TYPE QUESTIONS

Note : For an A.P. with the 1st term and common difference 'a' and 'd' respectively, we have :

(a) n^{th} term from the end = $(m - n + 1)$th term from the beginning, where m is the number of terms in the A.P.

$\Rightarrow$ n^{th} term from the end = $(a) + (m - n)d$

(b) If 'l' is the last term of the A.P., then

n^{th} term from the end is the n^{th} term of an A.P. whose first term is 'l' and common difference is '$-d$'

$\Rightarrow$ n^{th} term from the end = $l + (n - 1)(-d)$

Q. 1. *If 9th term of an A.P. is zero, prove that its 29th term is double of its 19th term.*

[NCERT Exemplar]

Sol. Let 'a' be the first term and 'd' be the common difference.

Now, Using $T_n = a + (n - 1)\, d$, we have

$T_9 = a + 8d \Rightarrow a + 8d = 0$ ...(1) [$\because T_9 = 0$ Given]

$T_{19} = a + 18d = (a + 8d) + 10d = (0) + 10d = 10d$ ...(2) [$\because a + 8d = 0$]

$T_{29} = a + 28d$

$= (a + 8d) + 20d$

$= 0 + 20d = 20d$ [$\because a + 8d = 0$]

$= 2 \times (10d) = 2\,(T_{19})$ [$\because T_{19} = 10d$]

$\Rightarrow \quad T_{29} = 2\,(T_{19})$

Thus, the 29th term of the A.P. is double of its 19th term.

Q. 2. *If $T_n = 3 + 4n$ then find the A.P. and hence find the sum of its first 15 terms.*

Sol. Let the first term be 'a' and the common difference be 'd'.

$\because \quad T_n = a + (n - 1)\, d$

$\therefore \quad T_1 = a + (1 - 1)\, d = a + 0 \times d = a$

$T_2 = a + (2 - 1)\, d = a + d$

But it is given that

$T_n = 3 + 4n$

$\therefore \quad T_1 = 3 + 4\,(1) = 7$

$\Rightarrow$ First term, $a = 7$

Also, $T_2 = a + d = 3 + 4\,(2) = 11$

$\therefore \quad d = T_2 - T_1 = 11 - 7 = 4$

Now, using $S_n = \frac{n}{2}\,[2a + (n - 1)\, d]$, we get

$S_{15} = \frac{15}{2}\,[2\,(7) + (15 - 1) \times 4]$

$= \frac{15}{2}\,[14 + 14 \times 4]$

$= \frac{15}{2}[70]$

$= 15 \times 35 = 525$

Thus, the sum of first 15 terms = **525.**

Q. 3. *Which term of the A.P.:*

3, 15, 27, 39, ... will be 120 more than its 53rd term?

Sol. The given A.P. is:

3, 15, 27, 39, ...

$\therefore \quad a = 3$

$d = 15 - 3 = 12$

$\therefore$ Using, $T_n = a + (n-1)\,d$, we have:

$$T_{53} = 3 + (53 - 1) \times 12$$
$$= 3 + (52 \times 12)$$
$$= 3 + 624 = 627$$

Now, $\quad T_{53} + 120 = 627 + 120 = 747.$

Let the required term be T_n

$\therefore \quad T_n = 747$

or $\quad a + (n-1)\,d = 747$

$\therefore \quad 3 + (n-1) \times 12 = 747$

$\Rightarrow \quad (n-1) \times 12 = 747 - 3 = 744$

$\Rightarrow \quad n - 1 = \dfrac{744}{12} = 62$

$\Rightarrow \quad n = 62 + 1 = 63$

Thus, the **63**rd term of the given A.P. is 120 more than its 53rd term.

Q. 4. *Find the 31st term of an A.P. whose 10th term is 31 and the 15th term is 66.*

Sol. Let the first term is '*a*' and the common difference is '*d*'.

Using $T_n = a + (n-1)\,d$, we have:

$T_{10} = a + 9d$

$\Rightarrow \quad 31 = a + 9d \quad$ ...(1)

Also $\quad T_{15} = a + 14d$

$\Rightarrow \quad 66 = a + 14d \quad$ ...(2)

Subtracting (1) from (2), we have:

$a + 14d - a - 9d = 66 - 31$

$\Rightarrow \quad 5d = 35$

$\Rightarrow \quad d = \dfrac{35}{5} = 7$

$\therefore$ From (1), $\quad a + 9d = 31$

$\Rightarrow \quad a + 9\,(7) = 31$

$\Rightarrow \quad a + 63 = 31$

$\Rightarrow \quad a = 31 - 63$

$\Rightarrow \quad a = -32$

Now, $\quad T_{31} = a + 30d$

$= -32 + 30\,(7)$

$= -32 + 210 = 178$

Thus, the 31st term of the given A.P. is **178.**

Q. 5. *If the 8th term of an A.P. is 37 and the 15th term is 15 more than the 12th term, find the A.P. Hence find the sum of the first 15 terms of the A.P.*

Sol. Let the 1st term = *a*

And the common difference = d

$\therefore$ Using $T_n = a + (n - 1) d$

$\therefore \quad T_8 = a + 7d$

$\Rightarrow \quad 37 = a + 7d$ ...(1)

Also $\quad T_{15} = a + 14d$

And $\quad T_{12} = a + 11d$

According to the question,

$$T_{15} = T_{12} + 15$$

$\Rightarrow \quad a + 14d = a + 11d + 15$

$\Rightarrow a - a + 14d - 11d = 15$

$\Rightarrow \quad 3d = 15 \quad \Rightarrow \quad d = \frac{15}{3} = 5$

From (1), we have:

$a + 7(5) = 37$

$\Rightarrow \quad a + 35 = 37$

$\Rightarrow \quad a = 37 - 35 = 2$

Since an, A.P. is given by :

$a, a + d, a + 2d, a + 3d, ...$

$\therefore$ The required A.P. is given by 2, 2 + 5, 2 + 2(5),...

2, 7, 12, ...

Now, using $S_n = \frac{n}{2}[2a + (n - 1) d]$

$\therefore \quad S_{15} = \frac{15}{2}[2(2) + 14 \times 5]$

$= \frac{15}{2}[4 + 70]$

$= \frac{15}{2} \times 74 = 15 \times 37 =$ **555.**

Q. 6. *The 5th and 15th terms of an A.P. are 13 and – 17 respectively. Find the sum of first 21 terms of the A.P.*

Sol. Let 'a' be the first term and 'd' be the common difference.

$\therefore$ Using $T_n = a + (n - 1) d$, we have:

$T_{15} = a + 14d = -17$ ...(1)

$T_5 = a + 4d = 13$ ...(2)

Subtracting (2) from (1), we have:

$(T_{15} - T_5) = -17 - 13 = -30$

$\Rightarrow a + 14d - a - 4d = -30$

$\Rightarrow \quad 10d = -30 \quad \Rightarrow \quad d = -3$

Substituting $d = -3$ in (2), we get

$a + 4d = 13$

$\Rightarrow \quad a + 4(-3) = 13$

$\Rightarrow \quad a + (-12) = 13$

$\Rightarrow \quad a = 13 + 12 = 25$

Now using $S_n = \frac{n}{2}$ [2a + (n – 1) d] we have:

$$S_{21} = \frac{21}{2} [2 (25) + (21 - 1) \times (-3)]$$

$$= \frac{21}{2} [50 + (-60)]$$

$$= \frac{21}{2} \times -10$$

$$= 21 \times (-5) = -105$$

Thus, the sum of first fifteen terms = **– 105.**

Q. 7. *The 1st and the last term of an A.P. are 17 and 350 respectively. If the common difference is 9 how many terms are there in the A.P.? What is their sum?*

Sol. Here, first term, $a = 17$

Last term $T_n = 350 = l$

∵ Common difference (d) = 9.

∴ Using $T_n = a + (n - 1) d$, we have:

$$350 = 17 + (n - 1) \times 9$$

$$\Rightarrow \quad n - 1 = \frac{350 - 17}{9}$$

$$= \frac{333}{9} = 37$$

$$\Rightarrow \quad n = 37 + 1 = 38$$

Thus, there are 38 terms.

Now, using, $S_n = \frac{n}{2} [a + l]$, we have

$$S_{38} = \frac{38}{2} [17 + 350]$$

$$= 19 [367] = 6973$$

Thus, the required sum of 38 terms = **6973.**

Q. 8. *If the sum of first 7 terms of an A.P. is 49 and that of first 17 terms is 289, find the sum of n terms.* (CBSE 2008 C)

$$_n = \frac{n}{2} [2a + (n - 1) d]$$

$$\therefore \quad S_7 = \frac{7}{2} [2a + 6d] = 49$$

$$\Rightarrow \quad \frac{7}{2} \times 2[a + 3d] = 49$$

$$\Rightarrow \quad 7 [a + 3d] = 49$$

$$\Rightarrow \quad a + 3d = \frac{49}{7} = 7$$

i.e., $a + 3d = 7$ ...(1)

Also $S_{17} = \frac{17}{2} [2a + 16d] = 289$

$\Rightarrow \quad \frac{17}{2} \times 2 \; [a + 8d] = 289$

$\Rightarrow \quad 17 \; [a + 8d] = 289$

$\Rightarrow \quad a + 8d = \frac{289}{17} = 17$

$\Rightarrow \quad a + 8d = 17$ ...(2)

Subtracting (2) from (1), we have:

$a + 8d - a - 3d = 17 - 7$

$\Rightarrow \quad 5d = 10 \quad \Rightarrow \quad d = 2$

From (1), we have

$a + 3\,(2) = 7$

$\Rightarrow \quad a + 6 = 7 \quad \Rightarrow \quad a = 7 - 6 = 1$

Now, $S_n = \frac{n}{2}\,[2a + (n - 1)\,d]$

$= \frac{n}{2}\,[2 \times 1 + (n - 1) \times 2]$

$= \frac{n}{2}\,[2 + 2n - 2]$

$= \frac{n}{2}[2n]$

Thus, the sum of n terms is $\boldsymbol{n^2}$.

Q. 9. *The first and last term of an A.P. are 4 and 81 respectively. If the common difference is 7, how many terms are there in the A.P. and what is their sum?*

Sol. Here, first term = 4 $\Rightarrow \; a = 4$ and $d = 7$.

Last term, $l = 81 \quad \Rightarrow \quad T_n = 81$

$\because \quad T_n = a + (n - 1)\,d$

$\therefore \quad 81 = 4 + (n - 1) \times 7$

$\Rightarrow \quad 81 - 4 = (n - 1) \times 7$

$\Rightarrow \quad 77 = (n - 1) \times 7 \quad \Rightarrow \quad n = \frac{77}{7} + 1 = 11 + 1 = 12$

$\Rightarrow$ There are 12 terms.

Now, using

$S_n = \frac{n}{2}\,(a + l)$

$\Rightarrow \quad S_{12} = \frac{12}{2}\,(4 + 81)$

$\Rightarrow \quad S_{12} = 6 \times 85 = 510$

$\therefore$ The sum of 12 terms of the A.P. is **510.**

Q. 10. *The angles of a quadrilateral are in A.P. whose common difference is 15°. Find the angles.*

Sol. Let one of the angles = a

$\because$ The angles are in an A.P.

$\therefore$ The angles are:

$a°, (a + d)°, (a + 2d)°$ and $(a + 3d)°$

$\because \quad d = 15$ [Given]

$\therefore$ The angles are:

a, $(a + 15)$, $[a + 2 (15)]$ and $[a + 3 (15)]$

i.e., a, $(a + 15)$, $(a + 30)$ and $(a + 45)$.

$\because$ The sum of the angles of a quadrilateral is 360°.

$\therefore$ $a + (a + 15) + (a + 30) + (a + 45) = 360°$

$\Rightarrow \quad 4a + 90° = 360°$

$\Rightarrow \quad 4a = 360° - 90° = 270°$

$\Rightarrow \quad a = \dfrac{270}{4} = 67\dfrac{1°}{2}$

$\therefore$ The four angles are:

$67\dfrac{1°}{2}, \left(67\dfrac{1}{2}+15\right)^{\circ}, \left(67\dfrac{1}{2}+30\right)^{\circ}$, and $\left(67\dfrac{1}{2}+45\right)^{\circ}$

or $\mathbf{67\dfrac{1°}{2}, 82\dfrac{1°}{2}, 97\dfrac{1°}{2}}$, and $\mathbf{112\dfrac{1°}{2}}$.

Q. 11. *The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.* [NCERT Exemplar]

Sol. Let a, b, c are the angles of the triangle, such that

$$c = 2a \quad ...(1)$$

Since a, b, c are in A.P.

Then $$b = \frac{a+c}{2} \quad ...(2)$$

From (1) and (2), we get

$a, \left(\dfrac{a+2a}{2}\right), 2a$ are the three angles of the triangle.

$\therefore \quad a+\left(\dfrac{a+2a}{2}\right)+2a = 180°$

$\Rightarrow \quad 2a + a + 2a + 4a = 360°$

$\Rightarrow \quad 9a = 360°$

$\Rightarrow \quad a = \dfrac{360°}{9} = 40°$

$\therefore$ The smallest angle $= 40°$

The greatest angle $= 2a = 2 \times 40° = 80°$

The third angle $= \dfrac{a+c}{2} = \dfrac{40+80}{2} = 60°$

Thus the angles of the triangle are : **40°, 60°, 80°**.

Q. 12. *Find the middle term of the A.P. 10, 7, 4, ..., – 62.* (AI CBSE 2009 C)

Sol. Here, $a = 10$

$d = 7 - 10 = -3$

$T_n = (-62)$

$\therefore$ Using $T_n = a + (n - 1)d$, we have

$-62 = 10 + (n - 1) \times (-3)$

$\Rightarrow \quad n - 1 = \dfrac{-62-10}{-3} = \dfrac{-72}{-3} = 24$

$\Rightarrow \quad n = 24 + 1 = 25$

$\Rightarrow$ Number of terms = 25

$\therefore$ Middle term = $\left(\frac{n+1}{2}\right)$th term

$= \frac{25+1}{2}$ th term

$=$ 13th term

Now $T_{13} = 10 + 12d$

$= 10 + 12(-3)$

$= 10 - 36 = -26$

Thus, the middle term = **– 26**.

Q. 13. *Find the sum of all three digit numbers which are divisible by 7.* (CBSE 2012)

Sol. The three digit numbers which are divisible by 7 are:

105, 112, 119, ..., 994.

It is an A.P. such that

$a = 105$

$d = 112 - 105 = 7$

$T_n = 994 = l$

$\because \quad T_n = a + (n-1) \times d$

$\therefore \quad 994 = 105 + (n-1) \times 7$

$\Rightarrow \quad n - 1 = \frac{994 - 105}{7} = \frac{889}{7} = 127$

$\Rightarrow \quad n = 127 + 1 = 128$

Now, using $S_n = \frac{n}{2}[a + l]$

We have $S_{128} = \frac{128}{2}[105 + 994]$

$= 64 [1099]$

$= 70336$

Thus, the required sum = **70336.**

Q. 14. *Find the sum of all the three digit numbers which are divisible by 9.* (AI CBSE 2009 C)

Sol. All the three digit numbers divisible by 9 are:

117, 126, ..., 999 and they form an A.P.

Here, $a = 108$

$d = 117 - 108 = 9$

$T_n = 999 = l$

Now, using $T_n = a + (n-1)d$, we have

$999 = 108 + (n-1)(9)$

$\Rightarrow \quad 999 - 108 = (n-1) \times 9$

$\Rightarrow \quad 891 = (n-1) \times 9$

$\Rightarrow \quad n - 1 = \frac{891}{9} = 99$

$\Rightarrow \quad n = 99 + 1 = 100$

Now, the sum of n term of an A.P. is given

$$S_n = \frac{n}{2}[a + l]$$

$$\therefore \quad S_{100} = \frac{100}{2}[108 + 999]$$

$$= 50\ [1107]$$

$$= 55350$$

Thus, the required sum is **55350.**

Q. 15. *Find the sum of all the three digit numbers which are divisible by 11.* (CBSE 2009 C)

Sol. All the three digit numbers divisible by 11 are 110, 121, 132, ..., 990.

Here, $a = 110$

$d = 121 - 110 = 11$

$T_n = 990$

$\therefore$ Using $T_n = a + (n - 1)\,d$, we have

$$990 = 110 + (n - 1) \times 11$$

$$\Rightarrow \quad n - 1 = \frac{990 - 110}{11} = 80$$

$$\Rightarrow \quad n = 80 + 1 = 81$$

Now, using $S_n = \frac{n}{2}[a + l]$, we have

$$S_{81} = \frac{81}{2}[110 + 990]$$

$$= \frac{81}{2}[1100]$$

$$= 81 \times 550 = 44550$$

Thus, the required sum = **44550.**

Q. 16. *The sum of first six terms of an AP is 42. The ratio of 10th term to its 30th term is 1 : 3. Calculate the first term and 13th term of A.P.*

Sol. $\because \quad S_6 = \frac{6}{2}\{2a + (6-1)d\} = 42$

$\therefore \quad 6a + 15d = 42$ ...(1)

Also, $(a_{10}) : (a_{30}) = 1 : 3$

or $\frac{a + 9d}{a + 29d} = \frac{1}{3}$

$\Rightarrow \quad 3(a + 9d) = a + 29d$

$\Rightarrow \quad 3a + 27d = a + 27d$

$\Rightarrow \quad 2a = 2d$

$\Rightarrow \quad a = d$ ...(2)

From (1) $\quad 6d + 15d = 42 \quad \Rightarrow \quad d = \mathbf{2}$

From (2) $\quad a = d \quad \Rightarrow \quad d = \mathbf{2}$

Now, $\quad a_{13} = a + 12d$

$= 2 + 12 \times 2 = \mathbf{26}$

Q. 17. *If S_n the sum of n terms of an A.P. is given by $S_n = 3n^2 - 4n$, find the nth term.* (CBSE 2012)

Sol. We have:

$$S_{n-1} = 3\,(n - 1)^2 - 4\,(n - 1)$$

$= 3(n^2 - 2n + 1) - 4n + 4$
$= 3n^2 - 6n + 3 - 4n + 4$
$= 3n^2 - 10n + 7$

$\because$ nth term $= S_n - S_{n-1}$
$= 3n^2 - 4n - [3n^2 - 10n + 7]$
$= 3n^2 - 4n - 3n^2 + 10n - 7$
$= \mathbf{6n - 7.}$

Q. 18. *The sum of 4th and 8th terms of an A.P. is 24, and the sum of 6th and 10th terms is 44. Find the A.P.* (CBSE 2009)

Sol. Let, the first term = a

Common difference be = d

$\therefore$ Using $T_n = a + (n - 1)d$, we have

$T_4 = a + 3d$
$T_6 = a + 5d$
$T_8 = a + 7d$
$T_{10} = a + 9d$

$\because$ $T_4 + T_8 = 24$

$\therefore$ $(a + 3d) + (a + 7d) = 24$

$\Rightarrow$ $2a + 10d = 24$

$\Rightarrow$ $a + 5d = 12$ [*Dividing by 2*] ...(1)

Also $T_6 + T_{10} = 44$

$\therefore$ $(a + 5d) + (a + 9d) = 44$

$\Rightarrow$ $2a + 14d = 44$

$\Rightarrow$ $a + 7d = 22$ [*Dividing by 2*] ...(2)

Subtracting (1) from (2), we have:

$(a + 7d) - (a + 5d) = 22 - 12$

$\Rightarrow$ $2d = 10 \Rightarrow d = 5$

From (1), $a + 5(5) = 12$

$\Rightarrow$ $a = 12 - 25 = -13$

Since, the A.P. is given by:

$a, a + d, a + 2d, ...$

$\therefore$ We have the required A.P. as:

$-13, (-13 + 5), [-13 + 2(5)], ...$

or **$-13, -8, -3, ...$**

Q. 19. *If S_n, the sum of first n terms of an A.P. is given by*

$S_n = 5n^2 + 3n$

Then find the nth term. (CBSE 2009)

Sol. $\because$ $S_n = 5n^2 + 3n$

$\therefore$ $S_{n-1} = 5(n-1)^2 + 3(n-1)$
$= 5(n^2 - 2n + 1) + 3(n - 1)$

$= 5n^2 - 10n + 5 + 3n - 3$

$= 5n^2 - 7n + 2$

Now, nth term = $S_n - S_{n-1}$

∴ The required nth term

$= [5n^2 + 3n] - [5n^2 - 7n + 2]$

$= \mathbf{10n - 2.}$

Q. 20. *The sum of 5th and 9th terms of an* A.P. *is 72 and the sum of 7th and 12th term of 97. Find the* A.P. (CBSE 2009)

Sol. Let 'a' be the 1st term and 'd' be the common difference of the A.P.

Now, using $T_n = a + (n - 1) d$, we have

$T_5 = a + 4d$

$T_7 = a + 6d$

$T_9 = a + 8d$

$T_{12} = a + 11d$

$\because \quad T_5 + T_9 = 72$

$\therefore \quad a + 4d + a + 8d = 72$

$\Rightarrow \quad 2a + 12d = 72$

$\Rightarrow \quad a + 6d = 36$ [*Dividing by 2*] ...(1)

Also $T_7 + T_{12} = a + 6d + a + 11d = 97$

$\Rightarrow \quad 36 + a + 11d = 97$ [From (1)]

$\Rightarrow \quad a + 11d = 97 - 36$

$\Rightarrow \quad a + 11d = 61$ ...(2)

Subtracting (1) from (2), we get

$a + 11d - a - 6d = 61 - 36$

$\Rightarrow \quad 5d = 25$

$$\Rightarrow \quad d = \frac{25}{5}$$

From (1), we have

$a + 11 (5) = 61$

$a + 55 = 61$

$\Rightarrow \quad a = 61 - 55 = 6$

Now, a_n A.P. is given by

$a, a + d, a + 2d, a + 3d, ...$

∴ The required A.P. is:

6, (6 + 5), [6 + 2 (5)], [6 + 3 (5)], ...

or **6, 11, 16, 24, ...**

Q. 21. *In an A.P. the sum of its first ten terms is –150 and the sum of its next term is –550. Find the A.P.*

Sol. Let the first term $= a$

And the common difference $= d$

$$\therefore \quad S_{10} = \frac{10}{2}[2a+(10-1)d] = -150$$

$$\Rightarrow \quad 10a + 45d = -150$$

$$\Rightarrow \quad 2a + 9d = -30 \quad \text{...(1)}$$

$\because$ The sum of next 10 terms (i.e. $S_{20} - S_{10}$) = –550

$$\therefore \quad \frac{20}{2}[2a+(20-1)d]-(-150) = -550$$

$$\Rightarrow \quad 20a + 190d + 150 = -550$$

$$\Rightarrow \quad 2a + 19d + 15 = -55$$

$$\Rightarrow \quad 2a + 19d = -55 - 15$$

$$\Rightarrow \quad 2a + 19d = -70 \quad \text{...(2)}$$

Subtracting (1) from (2), we get

$$\begin{aligned} 2a + 19d &= -70 \\ 2a + 9d &= -30 \\ - \quad - \quad &\quad + \\ \hline 10d &= -40 \end{aligned} \quad \Rightarrow \quad d = \frac{-40}{10} = -4$$

From (1), $\quad 2(a) + 9(-4) = -30 \quad$ or $\quad a = \frac{6}{2} = 3$

Thus, AP is $a, a + d, a + 2d$ ...

or $\quad 3, [3 + (-4)], [3 + 2(-4)], ...$

or $\quad 3, -1, -5, ...$

Q. 22. *Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term?*

(AI CBSE 2009)

Sol. Let the 1st term is 'a' and common difference = d

$\therefore \quad a = 3$ and $d = 15 - 3 = 12$

Now, using $T_n = a + (n - 1)\, d$

$$\therefore \quad T_{21} = 3 + (21 - 1) \times 12$$

$$= 3 + 20 \times 12$$

$$= 3 + 240 = 243$$

Let the required term be the nth term.

$\because$ nth term = 120 + 21st term

= 120 + 243 = 363

Now $\quad T_n = a + (n - 1)\, d$

$$\Rightarrow \quad 363 = 3 + (n - 1) \times 12$$

$$\Rightarrow \quad 363 - 3 = (n - 1) \times 12$$

$$\Rightarrow \quad n - 1 = \frac{360}{12} = 30$$

$$\Rightarrow \quad n = 30 + 1 = 31$$

Thus the required term is the **31st** term of the A.P.

Q. 23. *Which term of the A.P. 4, 12, 20, 28, ... will be 120 more than its 21st term?*

(AI CBSE 2009)

Sol. Here, $a = 4$

$d = 12 - 4 = 8$

Using $T_n = a + (n-1)d$

$\therefore \quad T_{21} = 4 + (21 - 1) \times 8$

$= 4 + 20 \times 8 = 164$

$\because$ The required nth term $= T_{21} + 120$

$\therefore$ nth term $= 164 + 120 = 284$

$\therefore \quad 284 = a + (n-1)d$

$\Rightarrow \quad 284 = 4 + (n-1) \times 8$

$\Rightarrow \quad 284 - 4 = (n-1) \times 8$

$\Rightarrow \quad n - 1 = \dfrac{280}{8} = 35$

$\Rightarrow \quad n = 35 + 1 = 36$

Thus, the required term is the **36th** term of the A.P.

Q. 24. *The sum of n terms of an A.P. is $5n^2 - 3n$. Find the A.P. Hence find its 10th term.* (CBSE 2008)

Sol. We have:

$S_n = 5n^2 - 3n$

$\therefore \quad S_1 = 5(1)^2 - 3(1) = 2$

$\Rightarrow$ First term $T_1 = (a) = 2$

$S_2 = 5(2)^2 - 3(2) = 20 - 6 = 14$

$\Rightarrow$ Second term $T_2 = 14 - 2 = 12$

Now the common difference $= T_2 - T_1$

$\Rightarrow \quad d = 12 - 2 = 10$

$\because$ An A.P. is given by

$a, (a + d), (a + 2d)$ .....

$\therefore$ The required A.P. is:

2, (2 + 10), [2 + 2 (10)], ...

$\Rightarrow$ 2, 12, 22, ...

Now, using $T_n = a + (n-1)d$, we have

$T_{10} = 2 + (10 - 1) \times 10$

$= 2 + 9 \times 10$

$= 2 + 90 =$ **92.**

Q. 25. *Find the 10th term from the end of the A.P.:*

8, 10, 12, ..., 126 (CBSE 2008)

Sol. Here, $a = 8$

$d = 10 - 8 = 2$

$T_n = 126$

Using $T_n = a + (n-1)d$

$\Rightarrow \quad 126 = 8 + (n-1) \times 2$

$\Rightarrow \quad n - 1 = \dfrac{126 - 8}{2} = 59$

$\Rightarrow \quad n = 59 + 1 = 60$

$\therefore \quad l = 60$

Now 10th term from the end is given by

$l - (10 - 1) = 60 - 9 = 51$

Now, $T_{51} = a + 50d$

$= 8 + 50 \times 2$

$= 8 + 100 = 108$

Thus, the 10th term from the end is **108.**

Q. 26. *The sum of n terms of an A.P. is $3n^2 + 5n$. Find the A.P. Hence, find its 16th term.* (CBSE 2012)

Sol. We have,

$S_n = 3n^2 + 5n$

$\therefore \quad S_1 = 3(1)^2 + 5(1)$

$= 3 + 5 = 8$

$\Rightarrow \quad T_1 = 8 \Rightarrow a = 8$

$S_2 = 3(2)^2 + 5(2)$

$= 12 + 10 = 22$

$\Rightarrow \quad T_2 = 22 - 8 = 14$

Now $d = T_2 - T_1 = 14 - 8 = 6$

$\because$ An A.P. is given by,

$a, (a + d), (a + 2d), ...$

$\therefore$ The required A.P. is:

$8, (8 + 6), [8 + 2(6)], ...$

$\Rightarrow 8, 14, 20, ...$

Now, using $T_n = a + (n - 1)d$, we hve

$T_{16} = a + 15d$

$= 8 + 15 \times 6 = 98$

Thus, the 16th term of the A.P. is 98.

Q. 27. *The sum of 4th and 8th terms of an A.P. is 24 and the sum of 6th and 10th terms is 44. Find the first three terms of the A.P.* (AI CBSE 2008)

Sol. Let the first term be 'a' and the common difference be 'd'.

Using $T_n = a + (n - 1)d$, we have

$T_4 = a + 3d, \quad T_6 = a + 5d$

$T_8 = a + 7d \quad \text{and} \quad T_{10} = a + 9d$

Since $T_4 + T_8 = 24$

$\therefore \quad a + 3d + a + 7d = 24$

$\Rightarrow \quad 2a + 10d = 24 \Rightarrow a + 5d = 12$ ...(1)

Also, $T_6 + T_{10} = 44$

$\therefore \quad a + 5d + a + 9d = 44$

$\Rightarrow \quad 2a + 14d = 44 \Rightarrow a + 7d = 22$ ...(2)

Subtracting (2) from (1), we get,

$a + 7d - a - 5d = 22 - 12$

$\Rightarrow \quad 2d = 10 \Rightarrow d = 5$

Now from (1),

$$a + 5(5) = 12$$

$$\Rightarrow \quad a + 25 = 12 \quad \Rightarrow \quad a = -13$$

$$\therefore \quad \text{First term } (T_1) = a + 0 = -13$$

$$\text{Second term } (T_2) = a + d$$

$$= -13 + 5 = -8$$

$$\text{Third term } T_3 = -a + 2d$$

$$= -13 + 10 = -3$$

Q. 28. *In an A.P., the first term is 8, nth term is 33 and sum of first n terms is 123. Find n and d, the common difference.* (AI CBSE 2009)

Sol. Here,

$$\text{First term } T_1 = 8 \quad \Rightarrow \quad a = 8$$

$$n\text{th term } T_n = 33 = l$$

$$\because \quad S_n = 123 \qquad [\textit{Given}]$$

$\therefore$ Using, $S_n = \frac{n}{2}[a + l]$, we have

$$S_n = \frac{n}{2}[8 + 33]$$

$$\Rightarrow \quad 123 = \frac{n}{2} \times 41$$

$$\Rightarrow \quad n = \frac{123 \times 2}{41} = 6$$

Now, $T_6 = 33$

$$\Rightarrow a + 5d = 33$$

$$\Rightarrow 8 + 5d = 33$$

$$\Rightarrow \quad 5d = 33 - 8 = 25$$

$$\Rightarrow \quad d = \frac{25}{5} = 5$$

Thus, $\boldsymbol{n = 6}$ and $\boldsymbol{d = 5}$.

Q. 29. *For what value of n are the nth terms of two A.P.'s 63, 65, 67, ... and 3, 10, 17, ... equal?* [NCERT Exemplar (AI CBSE 2009)]

Sol. **For the 1st A.P.**

$$a = 63$$

$$d = 65 - 63 = 2$$

$$\therefore \quad T_n = a + (n-1)d \quad \Rightarrow \quad T_n = 63 + (n-1) \times 2$$

For the 2nd A.P.

$$a = 3$$

$$d = 10 - 3 = 7$$

$$\therefore \quad T_n = a + (n-1)d \quad \Rightarrow \quad T_n = 3 + (n-1) \times 7$$

$$\because \quad [T_n \text{ of 1st A.P.}] = [T_n \text{ of 2nd A.P.}]$$

$$\therefore \quad 63 + (n-1) \times 2 = 3 + (n-1) \times 7$$

$\Rightarrow \quad 63 - 3 + (n - 1) \times 2 = (n - 1)\, 7$

$\Rightarrow \quad 60 + (n - 1) \times 2 - (n - 1) \times 7 = 0$

$\Rightarrow \quad 60 + (n - 1)\, [2 - 7] = 0$

$\Rightarrow \quad 60 + (n - 1) \times (-5) = 0$

$\Rightarrow \quad (n - 1) = \dfrac{-60}{-5} = 12$

$\Rightarrow \quad n = 12 + 1 = 13$

Thus, the required value of n is **13.**

Q. 30. *If m times the mth term of an A.P. is equal to n times the nth term, find the (m + n)th term of the A.P.* [(AI CBSE 2008), (CBSE 2012)]

Sol. Let the first term $(T_1) = a$ and the common difference be 'd'.

$\therefore \quad n\text{th term} = a + (n - 1)\, d$

And $\quad m\text{th term} = a + (m - 1)\, d$

Also, $\quad (m + n)\text{th term} = a + (m + n - 1)\, d$ ...(1)

$\because \quad m\,(m\text{th term}) = n\,(n\text{th term})$

$\therefore \quad m\,[a + (m - 1)\, d] = n\,[a + (n - 1)\, d]$

$\Rightarrow \quad ma + m\,(m - 1)\, d = na + n\,(n - 1)\, d$

$\Rightarrow \quad ma + (m^2 - m)\, d - na - (n^2 - n)\, d = 0$

$\Rightarrow \quad ma - \text{na} + (m^2 - m)\, d - (n^2 - n)\, d = 0$

$\Rightarrow \quad a\,[m - n] + [m^2 - m - n^2 + n]\, d = 0$

$\Rightarrow \quad a\,[m - n] + [(m^2 - n^2) - (m - n)]\, d = 0$

$\Rightarrow \quad a\,[m - n] + [(m + n)\,(m - n) - (m - n)]\, d = 0$

$\Rightarrow \quad a\,[m - n] + (m - n)\,[m + n - 1]\, d = 0$

Dividing throughout by $(m - n)$, we have:

$a + [m + n - 1]\, d = 0$

$\Rightarrow \quad a + [(m + n) - 1]\, d = 0$ ...(2)

$\Rightarrow \quad (m + n)\text{ th term} = 0$ [From (1) and (2)]

Q. 31. *In an A.P., the first term is 25, nth term is – 17 and sum of first n terms is 60. Find 'n' and 'd', the common difference.* (AI CBSE 2008)

Sol. Here, the first term $a = 25$

And the nth term $= -17 = l$

Using $T_n = a + (n - 1)\, d$, we have:

$-17 = 25 + (n - 1)\, d$

$\Rightarrow \quad (n - 1)\, d = -17 - 25 = -42$

$\Rightarrow \quad (n - 1)\, d = -42 = d = \left[\dfrac{-42}{n - 1}\right]$ ...(1)

Also, $\quad S_n = \dfrac{n}{2}\,[a + l]$

$\Rightarrow \quad 60 = \dfrac{n}{2}\,[25 + (-17)]$

$\Rightarrow \quad 60 = \dfrac{n}{2}[8]$

$\Rightarrow \quad 60 = 4n \quad \Rightarrow \quad n = \dfrac{60}{4} = 15$

From (1), we have

$$d = \frac{-42}{15-1} = \frac{-42}{14} = -3$$

Thus, $\boldsymbol{n = 15}$ and $\boldsymbol{d = -3}$

Q. 32. *In an A.P., the first term is 22, nth term is – 11 and sum of first n terms is 66. Find n and d, the common difference.* (AI CBSE 2008)

Sol. We have

1st term $(T_1) = 22 \Rightarrow a = 22$

Last term $(T_n) = -11 \Rightarrow l = -11$

Using, $S_n = \frac{n}{2}[a + l]$, we have:

$$66 = \frac{n}{2}[22 + (-11)]$$

$$\Rightarrow 66 \times 2 = n[11]$$

$$\Rightarrow n = \frac{66 \times 2}{11} = 12$$

Again using

$$T_n = a + (n-1)d$$

We have:

$$T_{12} = 22 + (12-1) \times d$$

$$-11 = 22 + 11d \qquad [\because n\text{th term} = -11]$$

$$\Rightarrow 11d = -22 - 11 = -33$$

$$\Rightarrow d = \frac{-33}{11} = -3$$

Thus, $\boldsymbol{n = 12}$ and $\boldsymbol{d = -3}$

III. HOTS QUESTIONS

Q. 1. *Find the '6th' term of the A.P. :*

$$\frac{2m+1}{m}, \frac{2m-1}{m}, \frac{2m-3}{m}, \ldots$$

Sol. Here, $a_1 = \frac{2m+1}{m}, \quad a_2 = \frac{2m-1}{m}$

$$\therefore \quad d = a_2 - a_1$$

$$= \frac{2m-1}{m} - \frac{2m+1}{m} = \frac{2m-1-2m-1}{m}$$

$$= \frac{(-2)}{m}$$

Now, $a_n = a + (n-1)d$

$$\Rightarrow a_n = \left[\frac{2m+1}{m}\right] + (n-1)\left[\frac{-2}{m}\right]$$

$$= \left[\frac{2m+1}{m}\right]+\left[\frac{-2n}{m}\right]-1\left[\frac{-2}{m}\right]$$

$$= \frac{2m+1}{m}-\frac{2n}{m}+\frac{2}{m}$$

$$= \frac{2m+1-2n+2}{m}$$

$$= \frac{2m-2n+3}{m}$$

Thus, the n^{th} term is $\left(\frac{2m-2n+3}{m}\right)$

Again, we have

$$a_n = \frac{2m-2n+3}{m}$$

$$\Rightarrow \quad a_6 = \frac{2m-2(6)+3}{m} = \frac{2m-12+3}{m}$$

$$= \frac{2m-9}{m}$$

i.e., the 6th term is $\left(\frac{2m-9}{m}\right)$

Q. 2. *If the numbers a, b, c, d and e form an A.P., then find the value of a – 4b + 6c – 4d + e*

Sol. We have the first term of A.P. as 'a'.

Let D be the common difference of the given A.P.,

Then :

$b = a + D, \; c = a + 2D, \; d = a + 3D$ *and* $e = a + 4D$

$$\left[\begin{array}{l}\because 2^{nd} \text{ term} = a + \text{ common difference} \\ \;\; 3^{rd} \text{ term} = a + 2 \text{ common difference} \quad \text{etc.}\end{array}\right.$$

$\therefore \quad a - 4b + 6c - 4d + e$

$= a - 4(a + D) + 6(a + 2D) - 4(a + 3D) + (a + 4D)$

$= a - 4a + 6a - 4a + a - 4D + 12D - 12D + 4D$

$= 8a - 8a + 16D - 16D = 0$

Thus, $\quad a - 4b + 6c - 4d + e = 0$

Q. 3. *If* $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ *is the arithmetic mean between 'a' and 'b', then, find the value of 'n'.*

Sol. *Note* : A.M., between 'a' and 'b' = $\frac{1}{2}(a + b)$

We know that :

A.M. between 'a' and 'b' = $\frac{a+b}{2}$

It is given that,

$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between 'a' and 'b'

$$\therefore \quad \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$$

By cross multiplication, we get :

$$2\left[a^{n+1}+b^{n+1}\right] = \left[a^n+b^n\right]\left[a+b\right]$$

$$\Rightarrow \quad 2a^{n+1} + 2b^{n+1} = a^{n+1} + ab^n + a^n b + b^{n+1}$$

$$\Rightarrow \quad 2a^{n+1} - a^{n+1} + 2b^{n+1} - b^{n+1} = ab^n + a^n b$$

$$\Rightarrow \quad a^{n+1} + b^{n+1} = ab^n + a^n b$$

$$\Rightarrow \quad a^{n+1} - a^n b = ab^n - b^{n+1}$$

$$\Rightarrow \quad a^n[a-b] = b^n[a-b]$$

$$\Rightarrow \quad \frac{a^n}{b^n} = \frac{(a-b)}{(a-b)} = 1$$

$$\Rightarrow \quad \frac{a^n}{b^n} = \left(\frac{a^n}{b^n}\right)^0 \qquad \because x^0 = 1$$

$$\Rightarrow \quad n = 0$$

Q. 4. *If p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term $\frac{1}{p}$, prove that the sum of the first 'pq' terms is $\frac{1}{2}[pq+1]$.*

Hint: p^{th} *term* $= \frac{1}{q} \Rightarrow a + (p-1)d = \frac{1}{q}$ ...(1)

q^{th} *term* $= \frac{1}{p} \Rightarrow a + (q-1)d = \frac{1}{p}$ ...(2)

Solving (1) and (2), $d = \frac{1}{pq}$ *and* $q = \frac{1}{pq}$

Using $S_n = \frac{n}{2}[2a+(n-1)d]$, *we get :*

$$S_{pq} = \frac{pq}{2}\left[\frac{2}{pq}+(pq-1)\times\frac{1}{pq}\right] \Rightarrow S_{pq} = \frac{1}{2}(pq+1)$$

Q. 5. *If* $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ *are in A.P., prove that* a^2, b^2, c^2 *are also in A.P.*

Hint: $$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

[using the fact, that in an A.P.

(2nd term – 1st term) = (3rd term – 2nd term)]

Q. 6. *Solve the equation :*

1 + 4 + 7 + 10 + ... + x = 287 [NCERT Exemplar]

Sol. Since, $\left.\begin{matrix} 4-1=3 \\ 7-4=3 \\ 10-7=3 \end{matrix}\right\} \Rightarrow 1, 4, 7, 10, ..., x$ form an A.P.

$\therefore \quad a = 1, \quad d = 3$ and $a_n = x$

$\therefore \quad a_n = a + (n - 1)d$

$\Rightarrow \quad x = 1 + (n - 1)\, 3$ or $x = 3n - 2$

Also, $S_n = \frac{n}{2}(a+l)$

$\Rightarrow \quad 287 = \frac{n}{2}(1+x)$

$\Rightarrow \quad 2(287) = n[1 + (3n - 2)]$

$\Rightarrow \quad 574 = n[3n - 1]$

$\Rightarrow \quad 3n^2 - n - 574 = 0$

Solving the above quadratic equation, we get

$$n = \frac{-(-1) \pm \sqrt{1+4\times3\times574}}{6} = \frac{1\pm\sqrt{6888}}{6}$$

or $n = \frac{1\pm 83}{6} \quad \Rightarrow \quad n = 14$ or $\frac{-41}{3}$

But, negative n is not desirable.

$\therefore \quad n = 14$

$x = 3n - 2$

Now, $x = 3(14) - 2 = 42 - 2 = 40$

Thus, $x = 40$

Q. 7. *Find three numbers in A.P. whose sum is 21 and their product is 231.*

Sol. Let the three numbers in A.P. are:

$$a - d, \quad a, \quad a + d$$

$\therefore \quad (a - d) + a + (a + d) = 21$

$\Rightarrow \quad a - d + a + a + d = 21$

or $3a = 21 \quad \Rightarrow \quad a = 7$

Also, $(a - d) \times a \times (a + d) = 231$

$\therefore \quad (7 - d) \times 7 \times (7 + d) = 231$

$\Rightarrow \quad (7 - d)(7 + d) \times 7 = 231$

$\Rightarrow \quad 7^2 - d^2 = \frac{231}{7} = 33$

$\Rightarrow \quad 49 - d^2 = 33$

or $d^2 = 49 - 33 = 16$

$\Rightarrow \quad d = \pm 4$

Now, when $d = 4$, then three numbers in AP are : $(7 - 4)$, 7, $(7 + 4)$ i.e. **3, 7, 11**.

When $d = -4$, then three numbers in AP are : $[7 - (-4)]$, 7, $[7 + (-4)]$

or **11, 7, 3**

TEST YOUR SKILLS

1. Find the value of 'p' if the numbers x, $2x + p$, $3x + p$ are three successive terms of the AP.

2. Find p and q such that: $2p$, $2p + q$, $p + 4q$, 35 are in AP

3. Find a, b and c such that the following numbers are in A.P. :

a, 7, b, 23, c [NCERT Exemplar]

Hint:

$$7 - a = b - 7 \Rightarrow a + b = 14$$
$$23 - b = b - 7 \Rightarrow 2b = 30 \Rightarrow b = 15$$
$$23 - b = c - 23 \Rightarrow c + b = 46 \Rightarrow c = 46 - b = 46 - 15 = 31$$

And $a = 14 - b = 14 - 15 = -1$

4. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP. [NCERT Exemplar]

5. If $\frac{4}{5}, a, \frac{12}{5}$ are three consecutive terms of an AP, find the value of a.

6. For what value of p, are $(2p - 1)$, 7 and $\frac{11}{2}p$ three consecutive terms of an AP?

7. If $(x + 2)$, $2x$, $(2x + 4)$ are three consecutive terms of an AP, find the value of x.

(CBSE 2012)

8. For what value of p are $(2p - 1)$, 13 and $(5p - 10)$ are three consecutive terms of an A.P.?

9. Find the 10th term from the end of the A.P. 4, 9, 14, ... 254.

10. Find the 6th term of the AP 54, 51, 48...

11. Find the 8th term from the end of the AP : 7, 10, 13, ..., 184.

12. Find the 16th term of the AP 3, 5, 7, 9, 11, ...

13. Find the 12th term of the AP:

14, 9, 4, –1, –6, ...

14. Find the middle term of the AP :

20, 16, ..., –180

15. Find the 6th term from the end of the A.P.

17, 14, 11, ..., (–40)

16. Find the middle term of the AP :

10, 7, 4, ..., (–62)

17. Which term of the AP : 24, 21, 18, 13, ... is the first negative term?

Hint: *The first negative term will be the term immediately less than 0. i.e.* $T_n < 0$.

$\Rightarrow \quad [a + (n - 1)d] < 0 \qquad$ Here, $a = 24$, $d = (21 - 24) = -3$

$\Rightarrow \quad 3n > 27 \quad \Rightarrow \quad n > 9 \quad \therefore \quad n = 10$

18. The 6th term of an AP is –10 and its 10th term is –26. Determine the 15th term of the A.P.

19. For what value of n are the nth terms of the following two APs the same:

13, 19, 25, ... and 69, 68, 67, ...

20. The 8th term of an AP is zero. Prove that its 38th term is triple its 18th term.

Hint:

$T_8 = 0 \quad \Rightarrow \quad a + 7d = 0 \quad \Rightarrow \quad a = -7d$

$T_{38} = a + 37d = -7d + 37d = 30d$

Also, $T_{18} = a + 17d = -7d + 17d = 10d$

$30d = 3 \times (10d) \quad \Rightarrow \quad T_{38} = 3 \times T_{18}$

21. For what value of n, the nth terms of the following two AP's are equal?

23, 25, 27, 29, ... and –17, –10, –3, 4, ... [NCERT Exemplar]

22. Which term of the AP : 5, 15, 25, ... will be 130 more than 31st term?

Hint: *Let* a_n *be the required term*

i.e. a_n *be 130 more than* a_{31}

$\Rightarrow \quad a_n - a_{31} = 130$

23. Which term of the AP : 3, 15, 27, 39, ... will be 130 120 more than its 64th term?

24. The 9th term of an AP is 499 and its 499th term is 9. Which of its term is equal to zero.

25. Determine A.P. whose fourth term is 18 and the difference of the ninth term from fifteenth term is 30.

26. How many natural numbers are there between 200 and 500 which are divisible by 7?

Hint: *200 | ... 203 ... –497 | ... 500*

⟵ Divisible by –7 ⟶

$\therefore \quad a = 203, \quad d = 7 \quad \text{and } a_n = 497$

$\Rightarrow \quad a + (n - 1)\,d = a_n \quad \Rightarrow 203 + (n - 1) \times 7 = 497$

27. How many multiples of 7 are there between 100 and 300?

28. Find the value of the middle term of the following A.P. : –11, –7, –3, ..., 49.

29. Find the value of the middle term of the following A.P. : –6, –2, 2, ..., 58.

30. How many two digit numbers are divisible by 3?

Hint: Here, $a = 12$, $d = 3$ and $a_n = 99$

31. If the 9th term of an AP is zero, show that 29th term is double the 19th term.

Hint:

$$\frac{a_{29}}{a_{19}} = \frac{a+(29-1)d}{a+(19-1)d} = 2$$

Also, $a+(9-1)d = 0$

$a+8d = 0$

$a = -8d$

$$\Rightarrow \frac{a+28d}{a+18d} = 2$$

$$\Rightarrow \frac{-8d+28d}{-8d+18d} = 2$$

$$\Rightarrow \frac{20d}{10d} = 2 \quad \Rightarrow \quad 20d = 20d$$

$$\Rightarrow a_{29} = a_{19}$$

32. If in an AP, the sum of its first ten terms is –80 and the sum of its next ten terms is –280. Find the AP.

33. If in an A.P.

$$a_n = 20 \quad \text{and} \quad S_n = 399$$

then find '*n*'

Hint:

$$a_n = a + (n - 1)d \quad \Rightarrow \quad (n - 1)d = 19$$

$$S_n = \frac{n}{2}[2a+(n-1)d] = 399$$

$$= \frac{n}{2}[2(1)+19] = 399 \quad \Rightarrow \quad n = 38$$

34. Find the sum of all natural numbers from 1 to 100.

35. The first and last terms of an AP are 4 and 81 respectively. If the common difference is 7, how many terms are there in the A.P. and what is their sum?

36. How many terms of A.P.

a, 17, 25, ...

must be taken to get a sum of 450?

37. Find the sum of first hundred even natural numbers which are multiples of 5.

38. Find the sum of the first 30 positive integers divisible by 6.

39. Find the sum of those integers from 1 to 500 which are multiples of 2 or 5.

[NCERT Exemplar]

Hint: *Multiples of 2 are : 2, 4, 6, 8, 10, 12, 14, 16, ..., 500.*

Multiples of 5 are : 5, 10, 15, 20, 25, 30, ..., 500.

Multiples of 2 as well as 5 : 10, 20, 30, 40, ..., 500.

∴ *The required sum*

= [*Sum of multiplies of 2*] + [*Sum of multiples of 5*] – [*Multiples of 2 as well as 5*]

40. If the *n*th term of an A.P. is $2n + 1$, find S_n of the A.P.

41. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P. [NCERT Exemplar]

42. If S_n denotes the sum of n-terms of A.P. whose common differences is d and first term is a find:

$$S_n - 2S_{n-1} + S_{n-2}$$ (CBSE 2012)

Hint: $a_n = S_n - S_{n-1}$

43. If the ratio of 11th term to 18th term of an A.P. is 2 : 3. Find the ratio of the 5th term to the 21st term and also the ratio of the sum of the first five terms to the sum of first 21 terms. (CBSE 2012)

44. If in an A.P. the first term is 2, the last term is 29 and sum of the terms is 155. Find the common difference of the A.P.

45. The sum of n terms of an A.P. is $\left[\frac{5n^2}{2}+\frac{3n}{2}\right]$. Find the 20th term.

46. If S_n denotes the sum of first n terms of an A.P., prove that

$$S_{30} = 3(S_{20} - S_{10})$$ [Al CBSE Foreign 2014]

ANSWERS

Test Your Skills

1. $p = 0$	**2.** $p = 10$, $q = 5$	**3.** $a = -1$, $b = 15$, $c = 31$	**4.** $k = 0$
5. $a = 8/5$	**6.** $p = 2$	**7.** $x = 6$	**8.** $p = 5$
9. 209	**10.** 69	**11.** 163	**12.** 33
13. –41	**14.** –80	**15.** –25	**16.** –26
17. $n = 10$	**18.** –46	**19.** $n = 9$	**21.** $n = 9$
22. 44th	**23.** 74th	**24.** 508	**25.** 3, 8, 13, 18, ...
26. 43	**27.** 28	**28.** 17; 21	**29.** 26
30. 30	**32.** 1, –1, –3, –5, –7...		**33.** 38
34. 5050	**35.** 12, 510	**36.** 10	**37.** 50500
38. 2790	**39.** 27250	**40.** $n(n + 2)$	**41.** 3, 7, 11, 15, ...
42. d	**43.** 1 : 3; 5 : 49	**44.** $d = 3$	**45.** 99