Areas Related to Circles

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 12.1

Unless stated otherwise, use $\pi = \frac{22}{7}$

Q. 1. *The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.*

Sol. We have, $r_1 = 19$ cm

$r_2 = 9$ cm

$\therefore$ Circumference of circle-I $= 2\pi r_1 = 2\pi (19)$ cm

Circumference of circle-II $= 2\pi r_2 = 2\pi (9)$ cm

Sum of the circumferences of circle-I and circle-II

$= 2\pi (19) + 2\pi (9) = 2\pi (19 + 9)$ cm $= 2\pi (28)$ cm

Let R be the radius of the circle-III.

$\therefore$ Circumference of circle-III $= 2\pi R$

According to the condition,

$$2\pi R = 2\pi (28)$$

$$\Rightarrow \quad R = \frac{2\pi (28)}{2\pi} = 28 \text{ cm}$$

Thus, the radius of the new circle = **28 cm.**

Q. 2. *The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.*

Sol. We have,

Radius of circle-I, $r_1 = 8$ cm

Radius of circle-II, r_2 = 6 cm

$\therefore$ Area of circle-I $= \pi r_1^2 = \pi (8)^2$ cm^2

Area of circle-II $= \pi r_2^2 = \pi (6)^2$ cm^2

Let the area of the circle-III be R

$\therefore$ Area of circle-III $= \pi R^2$

Now, according to the condition,

$$\pi r_1^2 + \pi r_2^2 = \pi R^2$$

i.e. $\pi (8)^2 + \pi (6)^2 = \pi R^2$

$\Rightarrow$ $\pi (8^2 + 6^2) = \pi R^2$

$\Rightarrow$ $8^2 + 6^2 = R^2$

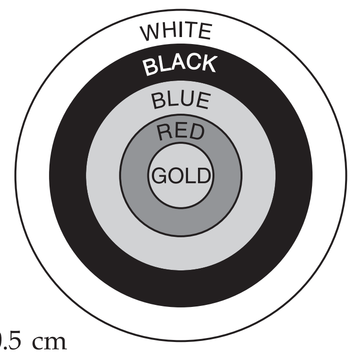
$\Rightarrow$ $64 + 36 = R^2$

$\Rightarrow$ $100 = R^2$

$\Rightarrow$ $10^2 = R^2 \Rightarrow R = 10$

Thus, the radius of the new circle = **10 cm.**

Q. 3. *Figure depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.*



Sol. Diameter of the innermost region = 21 cm

Radius of the innermost (Gold Scoring) region $= \frac{21}{2} = 10.5$ cm

$\therefore$ Area of Gold region $= \pi (10.5)^2$ cm^2

$$= \frac{22}{7} \times \left(\frac{105}{10}\right)^2 \text{cm}^2 = \frac{22}{7} \times \frac{105}{10} \times \frac{105}{10} \text{cm}^2$$

$$= \frac{22 \times 15 \times 105}{100} \text{cm}^2 = \mathbf{346.50\ cm^2}$$

Area of the Red region $= \pi (10.5 + 10.5)^2 - \pi (10.5)^2$

$= \pi (21)^2 - \pi (10.5)^2$

$$= \pi [(21)^2 - (10.5)^2] = \frac{22}{7} [(21 + 10.5)(21 - 10.5)] \text{ cm}^2$$

$$= \frac{22}{7} \times 31.5 \times 10.5 \text{ cm}^2 = 22 \times \frac{315}{10} \times \frac{15}{10} \text{cm}^2 = \mathbf{1039.5\ cm^2}$$

Area of Blue region $= \pi [(21 + 10.5)^2 - (21)^2]$ cm^2

$$= \frac{22}{7} [(31.5)^2 - (21)^2] \text{ cm}^2 = \frac{22}{7} [(31.5 + 21)(31.5 - 21)] \text{ cm}^2$$

$$= \frac{22}{7} \times 52.5 \times 10.5 \text{ cm}^2 = 22 \times \frac{75}{10} \times \frac{105}{10} \text{cm}^2 = \mathbf{1732.5\ cm^2}$$

Area of Black region $= \pi [(31.5 + 10.5)^2 - (31.5)^2]$ cm^2

$$= \frac{22}{7} [(42)^2 - (3.15)^2] \text{ cm}^2 = \frac{22}{7} [(42 - 31.5)(42 + 31.5)] \text{ cm}^2$$

$$= \frac{22}{7} \times 10.5 \times 73.5 \text{ cm}^2 = 22 \times \frac{15}{10} \times \frac{735}{10} \text{cm}^2 = \mathbf{2425.5\ cm^2}$$

Area of White region $= \pi [(42 + 10.5)^2 = (42)^2]$ cm^2

$$= \pi [(52.5)^2 - (42)^2] \text{ cm}^2 = \pi [(52.5 + 42) \times (52.5 - 42)]$$

$$= \frac{22}{7} \times 94.5 \times 10.5 = 22 \times \frac{945}{10} \times \frac{15}{10} = \mathbf{3118.5 \text{ cm}^2.}$$

Q. 4. *The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?*

Sol. Diameter of a wheel = 80 cm

$\therefore$ Radius of the wheel = $\frac{80}{2} = 40$ cm

$\therefore$ Circumference of the wheel

$$= 2\pi \times 40 = 2 \times \frac{22}{7} \times 40 \text{ cm}$$

$\Rightarrow$ Distance covered by a wheel in one revolution = $\frac{2 \times 22 \times 40}{7}$ cm

Distance travelled by the car in 1hr

$$= 66 \text{ km} = 66 \times 1000 \times 100 \text{ cm}$$

$\therefore$ Distance travelled in 10 minutes

$$= \frac{66 \times 1000 \times 100}{60} \times 10 \text{ cm} = 11 \times 100000 \text{ cm}$$

Now,

Number of revolutions

$$= \frac{[\text{Distance travelled in 10 minutes}]}{[\text{Distance travelled in one revolution}]}$$

$$= \frac{[1100000]}{\left[\frac{2 \times 22 \times 40}{7}\right]} = \frac{1100000 \times 7}{2 \times 22 \times 40} = 4375$$

Thus, the required number of revolutions = **4375.**

Q. 5. *Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is*

(A) 2 units *(B) π units* *(C) 4 units* *(D) 7 units*

Sol. We have:

[Numerical area of the circle] = [Numerical circumference of the circle]

$$\Rightarrow \quad \pi r^2 = 2\pi r$$

$$\Rightarrow \quad \pi r^2 - 2\pi r = 0$$

$$\Rightarrow \quad r^2 - 2r = 0$$

$$\Rightarrow \quad r(r - 2) = 0$$

$$r = 0 \quad \text{or} \quad r = 2$$

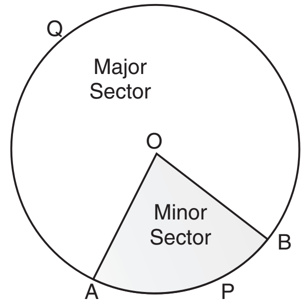
But r cannot be zero

$$\therefore \quad r = 2 \text{ units.}$$

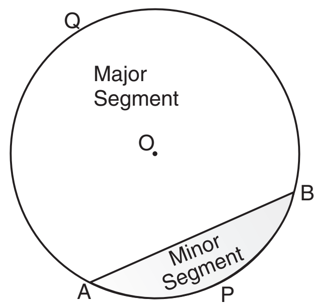
Thus, the option **(A) 2 units** is correct.

- **Area of Sector and Segment of a Circle**

The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a **sector** of the circle.



The portion (or part) of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle.



NOTE:

I. *∠AOB is called the '**angle of sector**'.*

II. *OAPB is the '**minor sector**' and OAQB is the '**major sector**'.*

III. *APB is the '**minor segment**' and AQB is the '**major segment**'.*

IV. *When we write '**sector**' and '**segment**' we will mean the '**minor-sector**' and the '**minor-segment**' respectively.*

Let us remember that

(*i*) Area of the sector of 'angle $\theta = \frac{\theta}{360°} \times \pi r^2$

(*ii*) Length of the arc of a sector of angle $\theta = \frac{\theta}{360°} \times 2\pi r$

(*iii*) Area of a segment = [Area of the corresponding sector] – [Area of the corresponding triangle]

NCERT TEXTBOOK QUESTIONS SOLVED

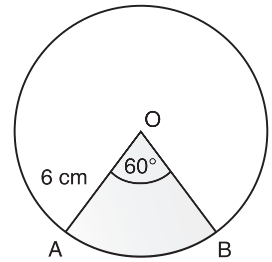
(EXERCISE 12.2)

Use $\pi = \frac{22}{7}$ (unless stated otherwise)

Q. 1. *Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.*

Sol. Here, $r = 6$ cm

$\theta = 60°$



$\therefore$ Using, the Area of a sector $= \frac{\theta}{360} \times \pi r^2$

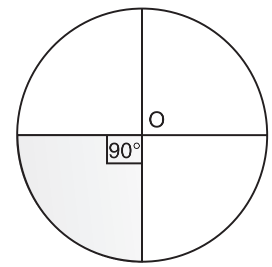
We have,

Area of the sector with $r = 6$ cm and $\theta = 60°$

$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2 = \frac{22}{7} \times 6 \text{ cm}^2 = \mathbf{\frac{132}{7} \text{ cm}^2}.$$

Q. 2. *Find the area of a quadrant of a circle whose circumference is 22 cm.*

Sol. Let radius of the circle = r



$\therefore \quad 2\pi r = 22$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow \quad r = 2 \times \frac{22}{7} \times \frac{1}{2} = \frac{7}{2}$ cm

Here $\theta = 90°$

$\therefore$ Area of the quadrant $\left(\frac{1}{4}\text{th}\right)$ of the circle,

$$= \frac{\theta}{360} \times \pi r^2 = \frac{90}{360°} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2 = \frac{1 \times 11 \times 7}{4 \times 2} \text{ cm}^2 = \mathbf{\frac{77}{8} \text{ cm}^2}.$$

Q. 3. *The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.*

Sol. [Length of minute hand] = [radius of the circle]

$\Rightarrow \quad r = 14$ cm

$\because$ Angle swept by the minute hand in 60 minutes = 360°

$\therefore$ Angle swept by the minute hand in 5 minutes $= \frac{360°}{60°} \times 5 = 30°$

Now, area of the sector with $r = 14$ cm and $\theta = 30°$

$$\frac{\theta}{360} \times \pi r^2 = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2 = \frac{11 \times 14}{3} \text{ cm}^2 = \mathbf{\frac{154}{3} \text{ cm}^2}$$

Thus, the required area swept by the minute hand by 5 minutes $= \mathbf{\frac{154}{3} \text{ cm}^2}$.

Q. 4. *A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding (i) minor segment (ii) major sector. (Use $\pi = 3.14$)*

Sol. Length of the radius (r) = 10 cm

Sector angle $\theta = 90°$

Area of the sector with $\theta = 90°$ and $r = 10$ cm

$$= \frac{90}{360} \times 10 \times 10 \times \frac{314}{100} \text{ cm}^2 = \frac{1}{4} \times 314 \text{ cm}^2 = \frac{157}{2} \text{ cm}^2 = 78.5 \text{ cm}^2$$

Now,

(*i*) Area of the minor segment

= [Area of minor sector] – [Area of rt. Δ *AOB*]

$$= [78.5 \text{ cm}^2] - \left[\frac{1}{2} \times 10 \times 10 \text{ cm}^2\right] = 78.5 \text{ cm}^2 - 50 \text{ cm}^2 = \mathbf{28.5 \text{ cm}^2}.$$

(*ii*) Area of major segment

= [Area of the circle] – [Area of the minor segment]

= $\pi r^2 - 78.5 \text{ cm}^2$

$= \left[\frac{314}{100} \times 10 \times 10 - 78.5\right] \text{cm}^2 = (314 - 78.5) \text{ cm}^2 = \mathbf{235.5 \text{ cm}^2}.$

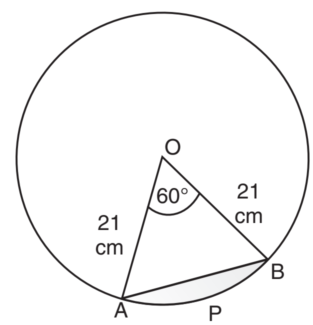
Q. 5. *In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:*

(*i*) *the length of the arc* [CBSE 2013]

(*ii*) *area of the sector formed by the arc*

(*iii*) *area of the segment formed by the corresponding chord*

Sol. Here, radius = 21 cm and $\theta = 60°$



(*i*) Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 21 \text{ cm} = 2 \times 22 \times 3 \text{ cm} = 132 \text{ cm}$

$\therefore$ Length of $\widehat{APB} = \frac{60}{360} \times 132 \text{ cm}$

$= \frac{1}{6} \times 132 \text{ cm} = \mathbf{22 \text{ cm}}$

(*ii*) Area of the sector with sector angle 60°

$= \frac{60°}{360°} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = \mathbf{11 \times 21 \text{ cm}^2 = 231 \text{ cm}^2}$

(*iii*) Area of the segment *APQ*

= [Area of the sector *AOB*] – [Area of Δ *AOB*] ...(1)

In ΔAOB, $OA = OB = 21$ cm

$\therefore$ $\angle A = \angle B = 60°$ [$\because \angle O = 60°$]

$\Rightarrow$ *AOB* is an equilateral Δ,

$\therefore$ $AB = 21$ cm

Draw $OM \perp AB$ such that

$$\frac{OM}{OA} = \sin 60° = \frac{\sqrt{3}}{2} \Rightarrow OM = 21 \times \frac{\sqrt{3}}{2} \text{ cm}$$

Now area of $\Delta OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21 \times 21 \times \frac{\sqrt{3}}{2} \text{ cm}^2$

$= \frac{441\sqrt{3}}{4} \text{ cm}^2$ ...(2)

From (1) and (2), we have:

$$\text{Area of segment} = [231 \text{ cm}^2] - \left[\frac{441\sqrt{3}}{4} \text{ cm}^2\right] = \mathbf{\left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2}.$$

Q. 6. *A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)*

Sol. Here, radius (r) = 15 cm

Sector angle $\theta = 60°$

$\therefore$ Area of the sector with $\theta = 60°$

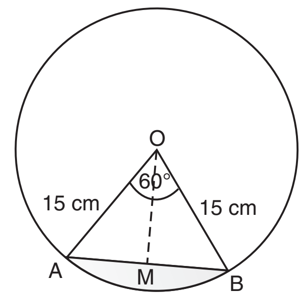
$$= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{314}{100} \times 15 \times 15 \text{ cm}^2 = \frac{11775}{100} \text{ cm}^2 = 117.75 \text{ cm}^2$$

Since $\angle O = 60°$ and $OA = OB = 15$ cm

$\therefore$ AOB is an equilateral triangle.

$\Rightarrow$ $AB = 15$ cm and $\angle A = 60°$

Draw $OM \perp AB$



$$\therefore \quad \frac{OM}{OA} = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad OM = OA \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2} \text{ cm}$$

Now, $\text{ar}(\Delta AOB) = \frac{1}{2} \times AB \times OM$

$$= \frac{1}{2} \times 15 \times 15 \frac{\sqrt{3}}{2} \text{ cm}^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2$$

$$= \frac{225 \times 1.73}{4} \text{ cm}^2 = 97.3125 \text{ cm}^2$$

Now area of the minor segment

= (Area of minor sector) – (ar Δ AOB)

= $(117.75 - 97.3125)$ cm^2 = **20.4375 cm^2**

Area of the major segment

= [Area of the circle] – [Area of the minor segment]

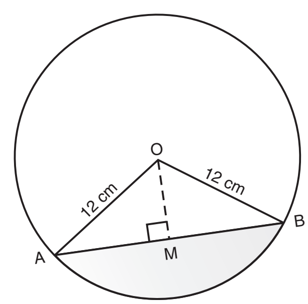
$$= \pi r^2 - 20.4375 \text{ cm}^2 = \left[\frac{314}{100} \times 15^2\right] - 20.4375 \text{ cm}^2$$

= $706.5 - 20.4375$ cm^2 = **686.0625 cm^2.**

Q. 7. *A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)*

Sol. Here, $\theta = 120°$ and $r = 12$ cm

$$\therefore \text{ Area of the sector} = \frac{\theta}{360°} \times \pi r^2$$



$$= \frac{120}{360} \times \frac{314}{100} \times 12 \times 12 \text{ cm}^2$$

$$= \frac{314 \times 4 \times 12}{100} \text{ cm}^2 = \frac{15072}{100} \text{ cm}^2 = 150.72 \text{ cm}^2 \quad ...(1)$$

Now, area of $\Delta AOB = \frac{1}{2} \times AB \times OM$ [$\because OM \perp AB$] ...(2)

In Δ OAB, $\angle O = 120°$

$\Rightarrow$ $\angle A + \angle B = 180° - 120 = 60°$

$\because$ $OB = OA = 12$ cm $\Rightarrow$ $\angle A = \angle B = 30°$

$$\text{So,} \quad \frac{OM}{OA} = \sin 30° = \frac{1}{2} \Rightarrow OM = OA \times \frac{1}{2}$$

$$\Rightarrow \quad OM = 12 \times \frac{1}{2} = 6 \text{ cm}$$

In right Δ AMO, $12^2 - 6^2 = AM^2$

$\Rightarrow$ $144 - 36 = AM^2$

$\Rightarrow$ $108 = AM^2$

$\Rightarrow \quad AM = \sqrt{108} = 6\sqrt{3}$

$\Rightarrow \quad 2\,AM = 12\sqrt{3}$

$\Rightarrow \quad AB = 12\sqrt{3}$ cm

Now, from (2),

$$\text{Area of } \Delta\, AOB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 12\sqrt{3} \times 6\ \text{cm}^2 = 36\sqrt{3}\ \text{cm}^2$$

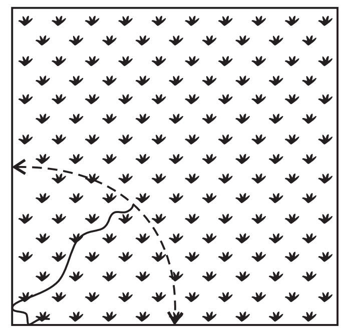
$$= 36 \times 1.73\ \text{cm}^2 = 62.28\ \text{cm}^2 \qquad ...(3)$$

From (1) and (3)

Area of the minor segment

= [Area of minor segment] – [Area of Δ AOB]

= [150.72 cm^2] – [62.28 cm^2] = **88.44 cm^2.**



Q. 8. *A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find:*

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Sol. Here, Length of the rope = 5 m

∴ Radius of the circular region grazed by the horse = 5 m

(*i*) Area of the circular portion grazed

$$= \frac{90°}{360°} \times \pi r^2 \qquad [\because\ \theta = 90° \text{ for a square field.}]$$

$$= \frac{90}{360} \times \frac{314}{100} \times 5 \times 5\ \text{m}^2 = \frac{1}{4} \times \frac{314}{16}\ \text{m}^2 = \frac{157}{8}\ \text{m}^2 = \mathbf{19.625\ m^2}$$

(*ii*) When length of the rope is increased to 10 m,

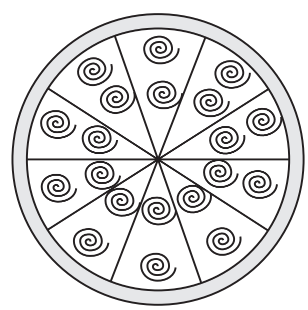
∴ $r = 10$ m

⇒ Area of the circular region where θ = 90°.

$$= \frac{\theta}{360°} \times \pi r^2 = \frac{90}{360} \times \frac{314}{100} \times (10)^2\ \text{m}^2 = \frac{1}{4} \times 314\text{m}^2 = 78.5\text{m}^2$$

∴ Increase in the grazing area

$$= 78.5 - 19.625\ \text{m}^2 = \mathbf{58.875\ m^2}.$$



Q. 9. *A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. Find:*

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

Sol. Diameter of the circle = 35 mm

$$\therefore \quad \text{Radius } (r) = \frac{35}{2}\ \text{mm}$$

(*i*) Circumference $= 2\pi r = 2 \times \frac{22}{7} \times \frac{35}{2}$ mm $= 22 \times 5 = 110$ mm

Length of 1 piece of wire used to make diameter to divide the circle into 10 equal sectors = 35 mm

∴ Length of 5 pieces = 5 × 35 = 175 mm

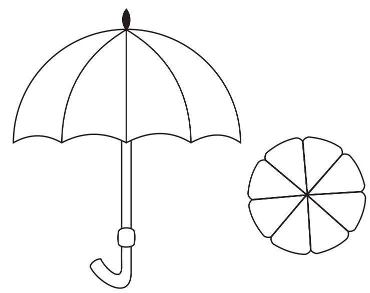
∴ Total length of the silver wire = 110 + 175 mm = **285 mm** (*ii*) Since the circle is divided into 10 equal sectors,

$$\therefore \quad \text{Sector angle } \theta = \frac{360^\circ}{10} = 36^\circ$$

$$\Rightarrow \quad \text{Area of each sector} = \frac{\theta}{360} \times \pi r^2 = \frac{36}{360} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{ mm}^2$$

$$= \frac{11 \times 35}{4} \text{ mm}^2 = \mathbf{\frac{385}{4} \text{ mm}^2}.$$

Q. 10. *An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.*



Sol. Here, radius (r) = 45 cm

Since circle is divided in 8 equal parts,

∴ Sector angle corresponding to each part

$$\theta = \frac{360^\circ}{8} = 45^\circ$$

⇒ Area of a sector (part)

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{45}{360} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2$$

$$= \frac{11 \times 45 \times 45}{4 \times 7} \text{ cm}^2 = \frac{22275}{28} \text{ cm}^2$$

∴ The required area between the two ribs = $\mathbf{\frac{22275}{28} \text{ cm}^2}$.

Q. 11. *A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.*

Sol. Here, radius (r) = 25 cm

Sector angle (θ) = 115°

∴ Area cleaned by each sweep of the blades

$$= \left[\frac{\theta}{360} \times \pi r^2\right] \times 2 \qquad [\because \textit{Each sweep will have to and fro movement}]$$

$$= \left[\frac{115}{360} \times \frac{22}{7} \times 25 \times 25\right] \times 2 \text{ cm}^2$$

$$= \frac{23 \times 11 \times 25 \times 25}{18 \times 7} \text{ cm}^2 = \mathbf{\frac{158125}{126} \text{ cm}^2}.$$

Q. 12. *To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)*

Sol. Here, Radius (r) = 16.5 km

Sector angle (θ) = 80°

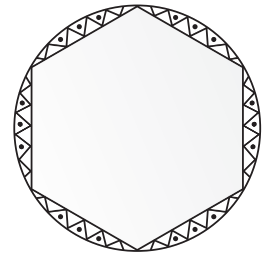
∴ Area of the sea surface over which the ships are warned

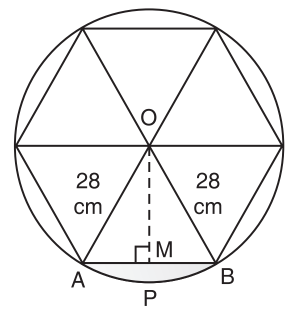
$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{80}{360} \times \frac{314}{100} \times \frac{165}{10} \times \frac{165}{10} \text{ km}^2$$

$$= \frac{157 \times 11 \times 11}{100} \text{ km}^2 = \frac{18997}{100} \text{ km}^2 = \mathbf{189.97 \text{ km}^2}.$$

Q. 13. *A round table cover has six equal designs as shown in Fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm². (Use $\sqrt{3}$ = 1.7)*





Sol. Here, $r = 28$ cm

Since, the circle is divided into six equal sectors.

$\therefore$ Sector angle $\theta = \dfrac{360°}{6} = 60°$.

$\therefore$ Area of the sector with $\theta = 60°$ and $r = 28$ cm

$$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 \text{ cm}^2$$

$$= \frac{44 \times 28}{3} \text{ cm}^2 = 410.67 \text{ cm}^2 \quad ...(1)$$

Now, area of 1 design

= Area of segment APB

= Area of sector – Area of $\Delta\, AOB$ ...(2)

In $\Delta\, AOB$, $\angle AOB = 60°$, $OA = OB = 28$ cm

$\therefore$ $\angle OAB = 60°$ and $\angle OBA = 60°$

$\Rightarrow$ $\Delta\, AOB$ is an equilateral triangle.

$\Rightarrow$ $AB = AO = BO$

$\Rightarrow$ $AB = 28$ cm

Draw $OM \perp AB$

$\therefore$ In right $\Delta\, AOM$, we have

$$\frac{OM}{OA} = \sin 60° = \frac{\sqrt{3}}{2} \Rightarrow OM = OA \times \frac{\sqrt{3}}{2} \text{ cm}$$

$$\Rightarrow \quad OM = 28 \times \frac{\sqrt{3}}{2} \text{ cm}$$

$$\Rightarrow \quad OM = 14\sqrt{3} \text{ cm}$$

$$\therefore \quad \text{Area of } \Delta\, AOB = \frac{1}{2}\, AB \times OM = \frac{1}{2} \times 28 \times 14\sqrt{3} \text{ cm}^2$$

$$= 14 \times 14\sqrt{3} \text{ cm}^2$$

$$= 14 \times 14 \times 1.7 \text{ cm}^2 = 333.3 \text{ cm}^2 \quad ...(3)$$

Now, from (1), (2) and (3), we have:

Area of segment APQ = 410.67 cm² – 333.2 cm² = 77.47 cm²

$\Rightarrow$ Area of 1 design = 77.47 cm²

$\therefore$ Area of the 6 equal designs = 6 × (77.47) cm²

= 464.82 cm²

Cost of making the design at the rate of ₹ 0.35 per cm²,

= ₹ 0.35 × 464.82

= **₹ 162.68.**

Q. 14. *Tick the correct answer in the following:*

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\dfrac{p}{180} \times 2\pi R$ (B) $\dfrac{p}{180} \times \pi R^2$ (C) $\dfrac{p}{360} \times 2\pi R$ (D) $\dfrac{p}{720} \times 2\pi R^2$

Sol. Here, radius $(r) = R$

Angle of sector $(\theta) = p°$

$\therefore$ Area of the sector $= \frac{\theta}{360} \times \pi r^2 = \frac{p}{360} \times \pi R^2 = \frac{2}{2} \times \left(\frac{p}{360} \times \pi R^2\right) = \frac{p \times 2\pi R^2}{720}$

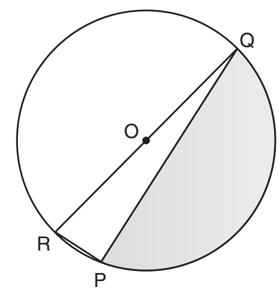
Thus, the option (*D*) $\mathbf{\frac{p}{720} \times 2\pi R^2}$ is correct.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 12.3

Unless stated otherwise, use $\pi = \frac{22}{7}$

Q. 1. *Find the area of the shaded region in Fig. PQ = 24 cm, PR = 7 cm and O is the centre of the circle.*



Sol. Since *O* is the centre of the circle,

$\therefore$ *QOR* is a diameter.

$\Rightarrow \quad \angle RPQ = 90°$ [Angle in a semi-circle]

Now, in right Δ *RPQ*,

$$RQ^2 = PQ^2 + PR^2$$

$$\Rightarrow \quad RQ^2 = 24^2 + 7^2 = 576 + 49 = 625$$

$$\Rightarrow \quad RQ = \sqrt{625} = 25$$

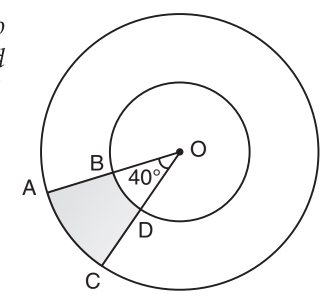
$\therefore$ Area of Δ $RPQ = \frac{1}{2}\ RQ \times RP = \frac{1}{2} \times 24 \times 7 \text{ cm}^2 = 12 \times 7 \text{ cm}^2 = 84 \text{ cm}^2$

Now, area of semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} = \frac{11 \times 625}{7 \times 4} \text{ cm}^2$

$$= \frac{6875}{28} \text{ cm}^2 = 245.54 \text{ cm}^2$$

$\therefore$ Area of the shaded portion $= 245.54 \text{ cm}^2 - 84 \text{ cm}^2 = 161.54 \text{ cm}^2$.

Q. 2. *Find the area of the shaded region in figures, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°.* *[AI. CBSE 2014]*



Sol. Radius of the outer circle = 14 cm

Here, $\theta = 40°$

$\therefore$ Area of the sector $AOC = \frac{40}{360} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$$= \frac{1}{9} \times 22 \times 2 \times 14 \text{ cm}^2 = \frac{616}{9} \text{ cm}^2$$

Radius of the inner circle = 7 cm

Here, also $\theta = 40°$

$\therefore$ Area of the sector *BOD*

$$= \frac{40}{360} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{1}{9} \times 22 \times 7 \text{ cm}^2 = \frac{154}{9} \text{ cm}^2$$

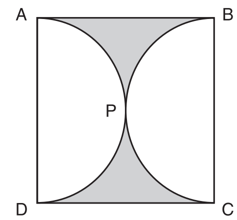
Now, area of the shaded region

$= [\text{Area of sector } AOC] - [\text{Area of sector } BOD]$

$= \frac{616}{9} - \frac{154}{9} \text{ cm}^2 = \frac{1}{9} [616 - 154] \text{ cm}^2 = \frac{1}{9} \times 462 \text{ cm}^2$

$= \mathbf{\frac{1}{3} \times 154 \text{ cm}^2.}$

Q. 3. *Find the area of the shaded region in figure, if ABCD is a square of side 14 cm and APD and BPC are semi-circles.* *[CBSE 2012]*



Sol. Side of the square = 14 cm

$\therefore$ Area of the square $ABCD$ = 14 × 14 cm^2 = 196 cm^2

Now, diameter of the circle = (Side of the square) = 14 cm

$\Rightarrow$ Radius of each of the circles = $\frac{14}{2} = 7$ cm

$\therefore$ Area of the semi-circle $APD = \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$

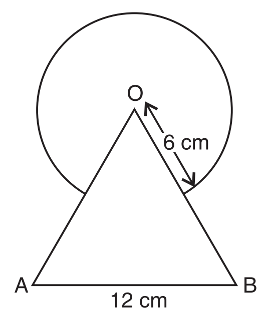
Area of the semi-circle BPC $= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$

$\therefore$ Area the shaded region

= [Area of the square] – [Area of semi-circle APD + Area of semi-circle BPC]

= 196 – [77 + 77] cm^2 = 196 – 154 cm^2 = **42 cm^2.**

Q. 4. *Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.* (CBSE 2012)

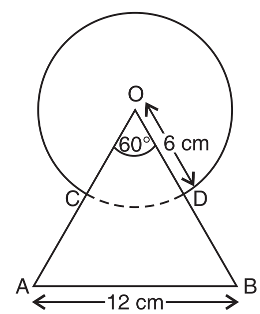


Sol. Area of the circle with radius = 6 cm.

$= \pi r^2 = \frac{22}{7} \times 6 \times 6 \text{ cm}^2 = \frac{792}{7} \text{ cm}^2$

Area of equilateral triangle, having side a = 12 cm, is given by

$$\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2 = 36\sqrt{3} \text{ cm}^2$$



$\because$ Each angle of an equilateral triangle = 60°

$\therefore \quad \angle AOB = 60°$

$\therefore \quad$ Area of sector $COD = \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

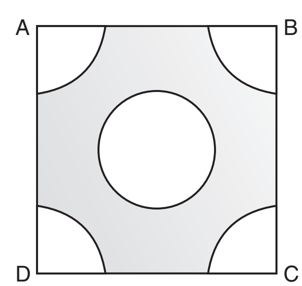
$= \frac{22 \times 6}{7} \text{ cm}^2 = \frac{132}{7} \text{ cm}^2$

Now, area of the shaded region,

= [Area of the circle] + [Area of the equilateral triangle] – [Area of the sector COD]

$= \frac{792}{7} + 36\sqrt{3} - \frac{132}{7} \text{ cm}^2 = \mathbf{\left[\frac{660}{7} + 36\sqrt{3}\right] \text{cm}^2}.$

Q. 5. *From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the square.* *[CBSE 2012]*



Sol. Side of the square = 4 cm

$\therefore$ Area of the square $ABCD$ = 4 × 4 cm^2 = 16 cm^2

∵ Each corner has a quadrant circle of radius 1 cm.

∴ Area of all the 4 quadrant squares

$$= 4 \times \frac{1}{4} \pi r^2 = \pi r^2 = \frac{22}{7} \times 1 \times 1 \text{ cm}^2 = \frac{22}{7} \text{ cm}^2$$

Diameter of the middle circle = 2 cm

⇒ Radius of the middle circle = 1 cm

∴ Area of the middle circle = πr^2

$$= \frac{22}{7} \times 7 \times 1 \text{ cm}^2 = \frac{22}{7} \text{ cm}^2$$

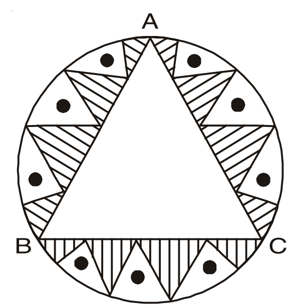
Now, area of the shaded region

= [Area of the square *ABCD*] – [(Area of the 4 quadrant circles) + (Area of the middle circle)]

$$= \left[16 \text{ cm}^2\right] - \left[\frac{22}{7} + \frac{22}{7} \text{ cm}^2\right] = 16 \text{ cm}^2 - 2 \times \frac{22}{7} \text{ cm}^2$$

$$= 16 \text{ cm}^2 - \frac{44}{7} \text{ cm}^2 = \frac{112 - 44}{7} \text{ cm}^2 = \mathbf{\frac{68}{7} \text{ cm}^2}.$$

Q. 6. *In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design.*

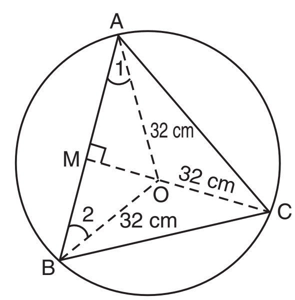


Sol. Area of the circle having radius r = 32 cm.

$$= \pi r^2$$

$$= \frac{22}{7} \times 32 \times 32 \text{ cm}^2 = \mathbf{\frac{22528}{7} \text{ cm}^2}$$

'*O*' is the centre of the circle,



∴ $AO = OB = OC = 32$ cm

⇒ $\angle AOB = \angle BOC = \angle AOC = 120°$

Now, in ΔAOB, $\angle 1 = 30°$

∵ $\angle 1 + \angle 2 = 60°$

Also $OA = OB \Rightarrow \angle 1 = \angle 2$

If $OM \perp AB$, then

$$\frac{OM}{OA} = \sin 30° = \frac{1}{2} \Rightarrow OM = OA \times \frac{1}{2}$$

$$\Rightarrow \quad OM = 32 \times \frac{1}{2} = 16 \text{ cm}$$

Also, $$\frac{AM}{AO} = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad AM = \frac{\sqrt{3}}{2} \times AO = \frac{\sqrt{3}}{2} \times 32$$

$$\Rightarrow \quad 2\,AM = AB = 2 \times \left(\frac{\sqrt{3}}{2} \times 32\right) = 32\sqrt{3} \text{ cm}$$

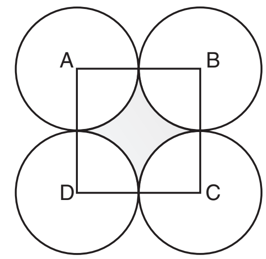
Now, area of ΔAOB, $= \frac{1}{2} \times OM \times AB = \frac{1}{2} \times 16 \times 32 \times \sqrt{3} = 256\sqrt{3} \text{ cm}^2$

Since area $\Delta ABC = 3 \times$ [area of ΔAOB] $= 3 \times 256 \times \sqrt{3} \text{ cm}^2 = \mathbf{768\sqrt{3} \text{ cm}^2}$

Now, area of the design = [Area of the circle] – [Area of the equilateral triangle]

$$= \mathbf{\left(\frac{22528}{7} - 768\sqrt{3}\right) \text{cm}^2}.$$

Q. 7. *In figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.* (CBSE 2012)



Sol. Side of the square $ABCD$ = 14 cm

$\therefore$ Area of the square $ABCD = 14 \times 14 \text{ cm}^2 = 196 \text{ cm}^2$.

$\because$ Circles touch each other

$\therefore$ Radius of a circle $= \dfrac{14}{2} = 7$ cm

Now, area of a sector of radius 7 cm and sector angle θ as 90°

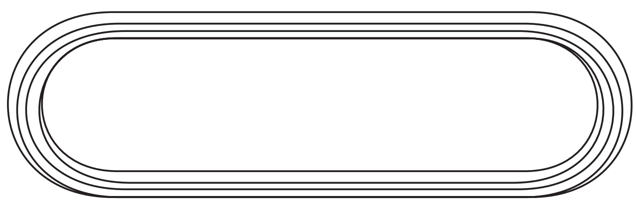
$$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{11 \times 7}{2} \text{ cm}^2$$

$\Rightarrow$ Area of 4 sectors $= 4 \times \left[\dfrac{11 \times 7}{2}\right] = 2 \times 11 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$

$\therefore$ Area of the shaded region = [Area of the square $ABCD$] – [Area of the 4 sectors]

$= 196 \text{ cm}^2 - 154 \text{ cm}^2 = \mathbf{42 \text{ cm}^2}$**.**

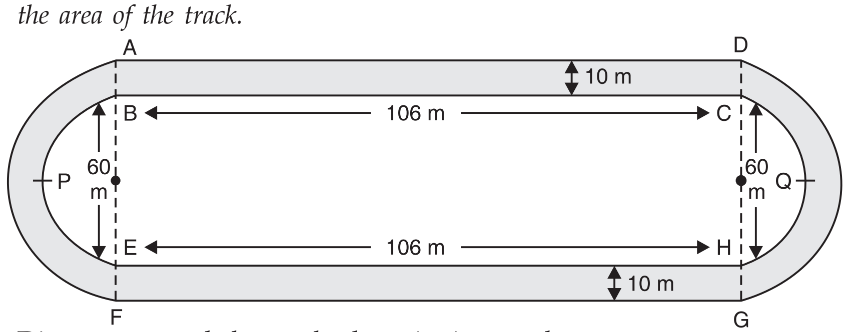
Q. 8. *The figure depicts a racing track whose left and right ends are semicircular.*



The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(*i*) *the distance around the track along its inner edge*

(*ii*) *the area of the track.* (CBSE 2012)



Sol. (*i*) Distance around the track along its inner edge

$= BC + EH + \widehat{BPE} + \widehat{CQH}$

$= 106 \text{ m} + 106 \text{ m} + \dfrac{1}{2}(2\pi r) + \dfrac{1}{2}(2\pi r)$

$= 212 \text{ m} + \dfrac{1}{2}\left(2 \times \dfrac{22}{7} \times 30\right) + \dfrac{1}{2}\left(2 \times \dfrac{22}{7} \times 30\right)$ $\quad \left[\therefore r = \dfrac{1}{2} BE = \dfrac{1}{2} \times 60 = 30 \text{ m}\right]$

$= 212 \text{ m} + \dfrac{1320}{7} \text{ m} = \dfrac{2804}{7} \text{ m}$

(*ii*) Now, area of the track

= Area of the shaded region

= (Area of rectangle *ABCD*) + (Area of rectangle *EFGH*)

$$+2\left[\begin{pmatrix}\text{Area of 2 semi-circles}\\ \text{each of radius 40 m}\end{pmatrix}-\begin{pmatrix}\text{Area of 2 sem-circles}\\ \text{each of radius 30 cm}\end{pmatrix}\right]$$ [$\because$ *The track is 10 m wide*]

$\Rightarrow$ Area of the track

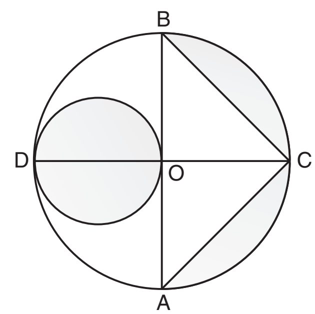
$$= (106 \times 10\ m^2) + (106 \times 10\ m^2) + 2\left[\frac{1}{2}\times\frac{22}{7}\times(40)^2 - \frac{1}{2}\times\frac{22}{7}\times(30)^2\right]m^2$$

$$= 1060\ m^2 + 1060\ m^2 - 2\left[\frac{1}{2}\times\frac{22}{7}\left(40^2 - 30^2\right)\right]m^2$$

$$= 2120\ m^2 + 2\times\frac{1}{2}\times\frac{22}{7}\ [(40 + 30) \times (40 - 30)]\ m^2$$

$$= 2120\ m^2 + \frac{22}{7} \times 70 \times 10\ m^2 = 2120\ m^2 + 2200\ m^2 = \mathbf{4320\ m^2.}$$

Q. 9. *In the figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.* (CBSE 2012, 2013)



Sol. *O* is the centre of the circle.

$OA = 7$ cm

$\Rightarrow$ $AB = 2\ OA = 2 \times 7 = 14$ cm

$OC = OA = 7$ cm

$\because$ *AB* and *CD* are perpendicular to each other

$\Rightarrow$ $OC \perp AB$

$\therefore$ Area $\Delta\ ABC = \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 14\text{ cm} \times 7\text{ cm} = \mathbf{49\ cm^2}$

Again $OD = OA = 7$ cm

$\therefore$ Radius of the small circle $= \frac{1}{2}(OD) = \frac{1}{2}\times 7 = \frac{7}{2}$ cm

$\therefore$ Area of the small circle $= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\text{ cm}^2 = \frac{11\times 7}{2} = \mathbf{\frac{77}{2}\ cm^2}$

Radius of the big circle $= \frac{14}{2}$ cm $= 7$ cm

$\therefore$ Area of semi-circle *OABC* $= \frac{1}{2}\left(\frac{22}{7}\times 7\times 7\right)\text{cm}^2 = \frac{11\times 7\times 7}{7}\text{ cm}^2$

$= 11 \times 7\text{ cm}^2 = 77\text{ cm}^2$

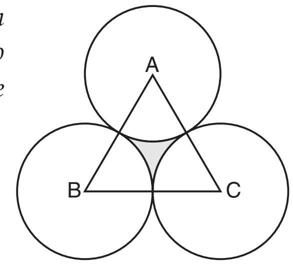
Now, Area of the shaded region

= [Area of the small circle] + [Area of the big semi-circle *OABC*] – [Area of Δ *ABC*]

$$= \frac{77}{2}\text{cm}^2 + 77\text{ cm}^2 - 49\text{ cm}^2 = \frac{77 + 154 - 98}{2}\text{cm}^2$$

$$= \frac{231 - 98}{2}\text{cm}^2 = \frac{133}{2}\text{cm}^2 = \mathbf{66.5\ cm^2.}$$

Q. 10. *The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig.). Find the area of the shaded region. (Use π = 3.14 and $\sqrt{3}$ = 1.73205).* (CBSE 2012)



Sol. Area of ΔABC = 17320.5 cm^2

$\because$ ΔABC is an equilateral triangle and area of an equilateral $\Delta = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$\therefore \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 17320.5$

$\Rightarrow \quad \frac{1.73205}{4}(\text{side})^2 = 17320.5$ $\quad [\because \sqrt{3} = 1.73205 \text{ (given)}]$

$\Rightarrow \quad \frac{173205}{400000}(\text{side})^2 = \frac{173205}{10}$

$\Rightarrow \quad (\text{side})^2 = \frac{173205}{10} \times \frac{400000}{173205}$

$\Rightarrow \quad (\text{side})^2 = 40000$

$\Rightarrow \quad (\text{side})^2 = (200)^2 \Rightarrow$ side = 200 cm

$\Rightarrow$ Radius of each circle = $\frac{200}{2}$ = 100 cm

Since each angle of an equilateral triangle is 60°,

$\therefore \quad \angle A = \angle B = \angle C = 60°$

$\therefore$ Area of a sector having angle of sector as 60° and radius 100 cm.

$= \frac{60}{360} \times \frac{314}{100} \times 100 \times 100 \text{ cm}^2 = \frac{1}{3} \times \frac{314}{100} \times 100 \times 100 \text{ cm}^2 = \frac{15700}{3} \text{ cm}^2$

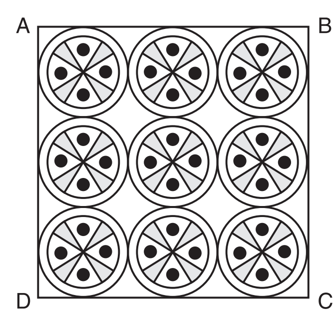
$\therefore$ Area of 3 equal sectors = $3 \times \frac{15700}{3} \text{ cm}^2$ = 15700 cm^2

Now, area of the shaded region

= [Area of the equilateral triangle *ABC*] – [Area of 3 equal sectors]

= 17320.5 cm^2 – 15700 cm^2 = **1620.5 cm^2.**

Q. 11. *On a square handkerchief, nine circular designs each of radius 7 cm are made (see figure). Find the area of the remaining portion of the handkerchief.*



Sol. $\because$ The circles touch each other.

$\therefore$ The side of the square *ABCD*

= 3 × diameter of a circle

= 3 × (2 × radius of a circle) = 3 × (2 × 7 cm)

= 42 cm

$\Rightarrow$ Area of the square *ABCD* = 42 × 42 cm^2 = **1764 cm^2.**

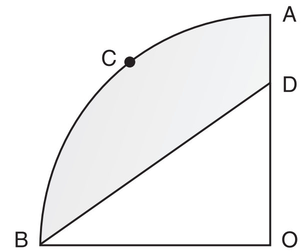
Now, area of one circle = $\pi r^2 = \frac{22}{7} \times 7 \times 7$ cm^2 = 154 cm^2

$\because$ There are 9 squares

$\therefore$ Total area of 9 circles = 154 × 9 = **1386 cm^2**

$\therefore$ Area of the remaining portion of the handkerchief = 1764 – 1386 cm^2 = **378 cm^2.**

Q. 12. *In the figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the*
(i) quadrant OACB, (ii) shaded region. (CBSE 2012)



Sol. Here, centre of the circle is O and radius = 3.5 cm.

$\therefore$ Area of the quadrant $OACB = \frac{1}{4}\pi r^2$

$$= \frac{1}{4}\times\frac{22}{7}\times\frac{35}{10}\times\frac{35}{10}\text{ cm}^2 = \frac{11}{2}\times\frac{35}{20} = \frac{11\times 7}{8}\text{ cm}^2 = \mathbf{\frac{77}{8}\ cm^2}$$

Now, ar ($\Delta\ BOD$) = $\frac{1}{2}\times OB\times OD$

$= \frac{1}{2}\times 3.5\times 2\text{ cm}^2$ [$\because$ *OB = 3.5 cm = radius and OD = 2 cm (given)*]

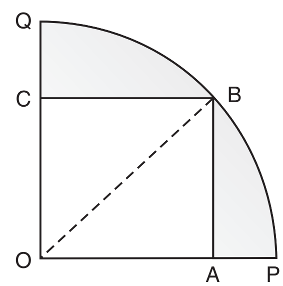
$= \frac{1}{2}\times\frac{35}{10}\times 2\text{ cm}^2 = \frac{7}{2}\text{ cm}^2$

$\therefore$ Area of the shaded region

= (Area of the quadrant $OACB$) – (Area of $\Delta\ BOD$)

$$= \left(\frac{77}{8}-\frac{7}{2}\right)\text{cm}^2 = \frac{77-28}{8}\text{ cm}^2 = \mathbf{\frac{49}{8}\ cm^2}.$$

Q. 13. *In the figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)*
(CBSE Sample Paper 2011) [CBSE (Delhi) 2014]



Sol. $OABC$ is a square such that its side OA = 20 cm.

$\therefore$ $OB^2 = OA^2 + OB^2$

$= [20^2 + 20^2] = [400 + 400] = [800]$

$\Rightarrow$ $OB = \sqrt{800} = 20\sqrt{2}$ cm.

$\Rightarrow$ Radius of the circle = $20\sqrt{2}$ cm.

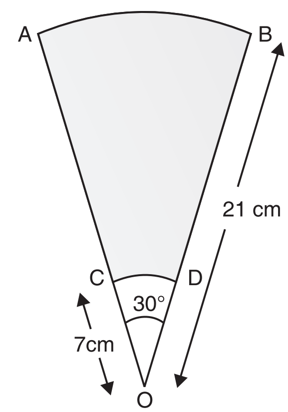
Now, area of the quadrant $OPBQ = \frac{1}{4}\pi r^2$

$$= \frac{1}{4}\times\frac{314}{100}\times 800\text{ cm}^2 = 314\times 2 = \mathbf{628\ cm^2}$$

Area of the square $OABC = 20\times 20\text{ cm}^2 = 400\text{ cm}^2$

$\therefore$ Area of the shaded region = 628 cm^2 – 400 cm^2 = **228 cm^2.**

Q. 14. *AB and CD are respectively areas of two concentric circles of radii 21 cm and 7 cm and centre O (see figure). If ∠AOB = 30°, find the area of the shaded region.* (CBSE 2012)



Sol. $\because$ Radius of bigger circle

R = 21 cm

and sector angle $\theta = 30°$

$\therefore$ Area of the sector $OAB = \frac{30}{360}\times\frac{22}{7}\times 21\times 21\text{ cm}^2$

$$= \frac{11\times 21}{2}\text{ cm}^2 = \frac{231}{2}\text{ cm}^2$$

Again, radius of the smaller circle

r = 7 cm

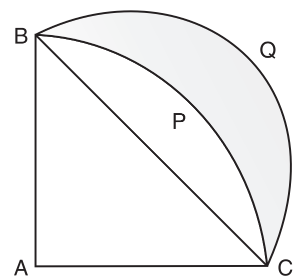
Here also, the sector angle is 30°

$\therefore$ Area of the sector $OCD = \frac{30}{360} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{77}{6} \text{ cm}^2$

$\therefore$ Area of the shaded region

$$= \frac{231}{2} - \frac{77}{6} \text{ cm}^2 = \frac{693 - 77}{6} \text{ cm}^2 = \frac{616}{6} \text{ cm}^2 = \mathbf{\frac{308}{3} \text{ cm}^2}.$$

Q. 15. *In the figure, ABPC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region.*



Sol. Radius of the quadrant = 14 cm

Therefore, area of the quadrant $ABPC$

$$= \left[\frac{90}{360} \times \frac{22}{7} \times 14 \times 4\right] \text{cm}^2 \quad \left[\text{using } \frac{\theta}{360} \times \pi r^2\right]$$

$= 22 \times 7 \text{ cm}^2 = \mathbf{154 \text{ cm}^2}$

Area of right $\Delta ABC = \frac{1}{2} \times 14 \times 14 \text{ cm}^2 = \mathbf{98 \text{ cm}^2}$

$\Rightarrow$ Area of segment $BPC = 154 \text{ cm}^2 - 98 \text{ cm}^2 = 56 \text{ cm}^2$

Now, in right ΔABC,

$$AC^2 + AB^2 = BC^2$$

$$\Rightarrow \quad 14^2 + 14^2 = BC^2$$

$$\Rightarrow \quad 196 + 196 = BC^2$$

$$\Rightarrow \quad BC^2 = 392 \quad \Rightarrow \quad BC = 14\sqrt{2} \text{ cm}.$$

$\therefore$ Radius of the semi-circle $BQC = \frac{14\sqrt{2}}{2} \text{ cm} = 7\sqrt{2} \text{ cm}$

$\therefore$ Area of the semi-circle $BQC = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} = 11 \times \sqrt{2} \times 7 \times \sqrt{2} \text{ cm}^2$$

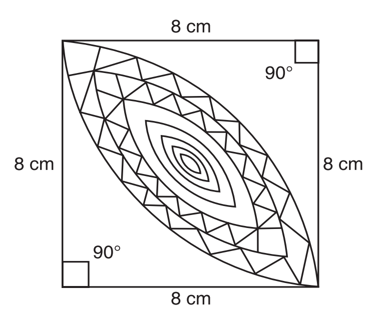
$$= 11 \times 7 \times 2 \text{ cm}^2 = 154 \text{ cm}^2$$

Now, area of the shaded region

= [Area of segment BQC] – [Area of segment BPC]

= $154 \text{ cm}^2 - 56 \text{ cm}^2 = \mathbf{98 \text{ cm}^2}$.

Q. 16. *Calculate the area of the designed region in the figure, common between the two quadrants of circles of radius 8 cm each.*



Sol. $\because$ Side of the square = 8 cm

$\therefore$ Area of the square $(ABCD)$ $= 8 \times 8 \text{ cm}^2$

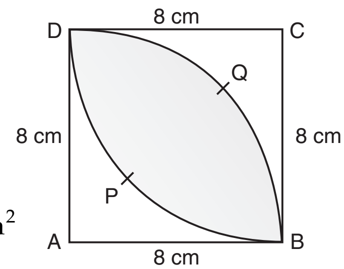
$= \mathbf{64 \text{ cm}^2}$

Now, radius of the quadrant $ADQB$ = 8 cm

$\therefore$ Area of the quadrant $ADQB = \frac{90}{360} \times \frac{22}{7} \times 8^2 \text{ cm}^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 64 \text{ cm}^2$$

$$= \frac{22 \times 16}{7} \text{ cm}^2$$



Similarly, area of the quadrant *BPDC* $= \frac{22 \times 16}{7} \text{ cm}^2$

$\therefore$ Sum of the two quadrants $= 2\left[\frac{22 \times 16}{7}\right] \text{cm}^2 = \frac{704}{7} \text{ cm}^2$

Now, area of design

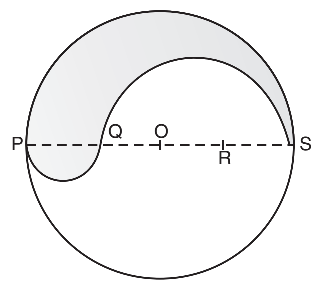
= [Sum of the areas of the two quadrants] – [Area of the square *ABCD*]

$$= \frac{704}{7} \text{ cm}^2 - 64 \text{ cm}^2 = \frac{704 - 448}{7} \text{ cm}^2 = \mathbf{\frac{256}{7} \text{ cm}^2}.$$

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *PQRS is a diameter of a circle of radius 6 cm. The equal lengths PQ, QR and RS are drawn on PQ and QS as diameters, as shown in figure. Find the perimeter of the shaded region.*



Sol. Diameter *PS* = 12 cm [$\because$ *Radius OS = 6 cm*]

Since *PQ, QR* and *RS* are three equal parts of diameter,

$\therefore$ $PQ = QR = RS = \frac{12}{3} \text{ cm} = 4 \text{ cm}$

$\Rightarrow$ $QS = 8 \text{ cm}$ [$\because$ $QS = 2 \times QR$]

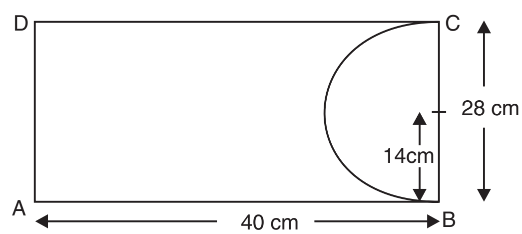
Now, the total required perimeter

$$= \widehat{PS} + \widehat{QS} + \widehat{PQ}$$

$$= \frac{1}{2}(2\pi \times 6) + \frac{1}{2}(2\pi \times 4) + \frac{1}{2}(2\pi \times 2) = 6\pi + 4\pi + 2\pi$$

$= \mathbf{12\pi}$ **cm.**

Q. 2. *A sheet of paper is in the form of a rectangle ABCD in which AB = 40 cm, and BC = 28 cm. A semi-circlular portion with BC as diameter is cut off. Find the area of the remaining paper.*



Sol. Length of the paper = 40 cm, width of the paper = 28 cm.

$\therefore$ Area of the rectangle = length × breadth $= 40 \times 28 \text{ cm}^2 = 1120 \text{ cm}^2$

Again, diameter of semi-circle = 28 cm.

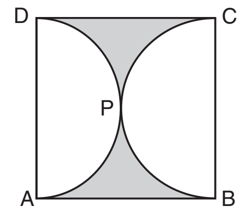
$\Rightarrow$ Radius of the semi-circle $= \frac{28}{2} = 14$ cm.

$\therefore$ Area of the semi-circle

$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2 = 11 \times 2 \times 14 \text{ cm}^2 = 308 \text{ cm}^2$$

$\therefore$ Area of the remaining paper $= 1120 \text{ cm}^2 - 308 \text{ cm}^2 =$ **812 cm²**.

Q. 3. *Find the area of the shaded region in the figure, if ABCD is a square of side 14 cm and APD and BPC are semi-circles.*



Sol. Side of the square = 14 cm

$\therefore$ Area of the square = 14 × 14 cm^2 = 196 cm^2

Also, diameter of each semi-circle = side of the square = 14 cm.

$$\Rightarrow \quad \text{Radius} = \frac{14}{2} = 7 \text{ cm.}$$

$$\text{Area of 1 semi-circle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 11 \times 7 \text{ cm}^2 = 77 \text{ cm}^2$$

$\Rightarrow$ Area of both semi-circles = 2 × 77 cm^2 = 154 cm^2.

Now, the area of the shaded region

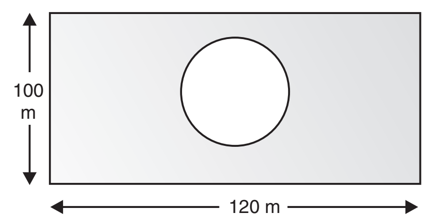
= [Area of the square *ABCD*] – [Area of both the semi-circles]

= (196 – 154) cm^2 = **42 cm^2.**

Q. 4. *A park is in the form of a rectangle 120 m long and 100 m wide. At the centre, there is a circular lawn of radius $\sqrt{1050}$ m. Find the area of the park excluding the lawn.* $\left[\text{Take } \pi = \frac{22}{7}\right]$

Sol. Length of the park l = 120 m

Breadth of the park b = 100 m



$\therefore$ Area of the park = $l \times b$ = 120 × 100 m^2

= 12000 m^2

Now, radius of the lawn = $\sqrt{1050}$ m

$\therefore$ Area of the lawn = πr^2

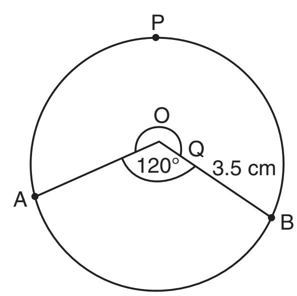
$$= \frac{22}{7} \times \left(\sqrt{1050}\right)^2 \text{ m}^2 = \frac{22}{7} \times 1050 \text{ m}^2$$

= 22 × 150 m^2 = 3300 m^2

$\therefore$ Area of the park excluding the central park

= 12000 – 3300 m^2 = **8700 m^2.**

Q. 5. *In the given figure, OAPB is a sector of a circle of radius 3.5 cm with the centre at O and ∠AOB = 120°. Find the length of OAPBO.*



Sol. Here, the major sector angle is given by

$$\theta = 360° - 120° = 240°$$

$$\text{Radius} = 3.5 \text{ cm} = \frac{35}{10} \text{ cm} = \frac{7}{2} \text{ cm}$$

$\therefore$ Circumference of the sector *APB*

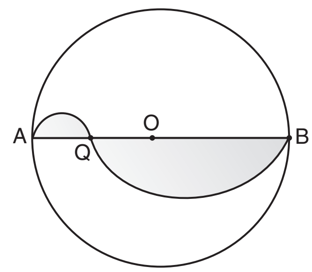
$$= \frac{\theta}{360} \times 2\pi r = \frac{240}{360} \times 2 \times \frac{22}{7} \times \frac{7}{2} \text{ cm} = \frac{2}{3} \times 22 \text{ cm} = \frac{44}{3} \text{ cm}$$

$\therefore$ Perimeter of *OAPBO*

= [Circumference of sector *AOB*] + *OA* + *OB*

$$= \frac{44}{3} + \frac{7}{2} + \frac{7}{2} \text{ cm} = \frac{44}{3} + 7 = \frac{44+21}{3} = \frac{65}{3} \text{ or } \mathbf{21\frac{2}{3} \text{ cm}}.$$

Q. 6. *Find the area of the shaded region of the following figure, if the diameter of the circle with centre O is 28 cm and* $AQ = \frac{1}{4}AB$.



Sol. We have $AB = 28$ cm

$\therefore \quad AQ = \frac{1}{4}AB = \frac{1}{4} \times 28\text{ cm} = 7\text{ cm}$

$\Rightarrow \quad BQ = 28 - 7 = 21$ cm

$\therefore$ Area of the semi-circle having diameter as 21 cm

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{2.1}{2} \times \frac{21}{2}\text{cm}^2 = \frac{11 \times 3 \times 21}{4}\text{cm}^2$$

$\therefore$ Also area of the semi-circle having diameter as 7 cm

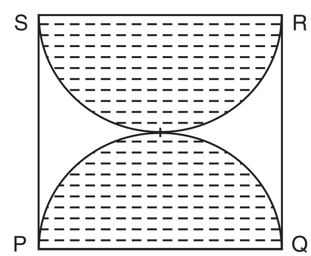
$$= \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\text{ cm}^2 = \frac{11 \times 7}{4}\text{ cm}^2$$

Thus the area of the shaded region

$$= \left[\frac{11 \times 3 \times 21}{4} + \frac{11 \times 7}{4}\right]\text{cm}^2 = \frac{11}{4}\ [3 \times 21 + 7]\text{ cm}^2$$

$$= \frac{11}{4} \times [63 + 7]\text{ cm}^2 = \frac{11}{4} \times 70 = \frac{770}{4}\text{cm}^2 = \mathbf{192.5\ cm^2.}$$

Q. 7. *PQRS is a square land of side 28 m. Two semi-circular grass covered postions are to be made on two of its opposite sides as shown in the figure. How much area will be left uncovered?* $\left[\textit{Take } \pi = \frac{22}{7}\right]$



Sol. Side of the square = 28 m

$\therefore$ Area of the square $PQRS = 28 \times 28\text{ m}^2$

Diameter of a semi-circle = 28 m

$\Rightarrow$ Radius of a semi-circle $= \frac{28}{2} = 14$ m

$\therefore$ Area of 1 semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 = 22 \times 14\text{ m}^2 = 308\text{ cm}^2$

$\Rightarrow$ Area of both the semi-circles

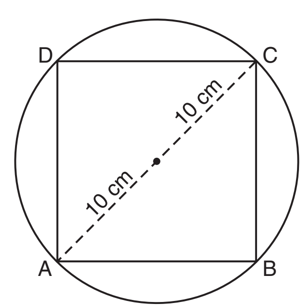
$= 2 \times 308\text{ m}^2 = 616\text{ m}^2$

$\therefore$ Area of the square left uncovered

$= (28 \times 28) - 616\text{ m}^2 = 784 - 616\text{ m}^2 = \mathbf{168\ m^2.}$

Q. 8. *Find the area of a square inscribed in a circle of radius 10 cm.*

Sol. Let *ABCD* be the square such that



$AB = BC = 10$ cm

$\therefore \quad AC^2 = AB^2 + BC^2$

$AB^2 + BC^2 = (10 \times 2)$

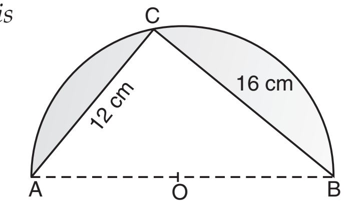
$\Rightarrow \quad x^2 + x^2 = (20)^2 \qquad$ [Let $AB = BC = x$]

$\Rightarrow \quad 2x^2 = 400$

$\Rightarrow \quad x^2 = \frac{400}{2} = 200\text{ cm}^2$

$\therefore$ Area of the square = $\mathbf{200\ cm^2.}$

Q. 9. *In the given figure, O is the centre of a circular arc and AOB is a straight line. Find the perimeter of the shaded region.*



Sol. O is the centre of the circle.

$\therefore$ AB is its diameter.

In right $\Delta\ ABC$,

$$AC^2 + BC^2 = AB^2$$

$$\Rightarrow \quad 12^2 + 16^2 = AB^2$$

$$\Rightarrow \quad 144 + 256 = AB^2 \quad \Rightarrow \quad AB^2 = 400$$

$$\Rightarrow \quad AB = \sqrt{400} = 20\text{cm}$$

$\therefore$ Circumference of semi-circle ACB

$$= \frac{1}{2}(2\pi r) = \frac{1}{2} \times 2 \times \frac{22}{7} \times \frac{20}{2}\text{ cm} = \frac{11 \times 20}{7}\text{ cm} = \frac{220}{7}\text{ cm}$$

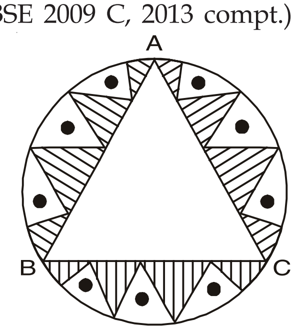
$\therefore$ Perimeter of the shaded region

$$= \frac{220}{7}\text{ cm} + 12\text{ cm} + 16\text{ cm} = 31.43\text{ cm} + 12\text{cm} + 16\text{cm} = \mathbf{59.43\ cm}$$

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *In a circular table cover of radius 70 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the figure. Find the total area of the design.*

[Use $\sqrt{3}$ *= 1.73 and* $\pi = \frac{22}{7}$*]* (AI CBSE 2009 C, 2013 compt.)



Sol. $\because$ Radius of the circle = 70 cm and O is the centre of the circle.

$\therefore \quad OA = OB = OC$

Since ΔABC is an equilateral triangle,

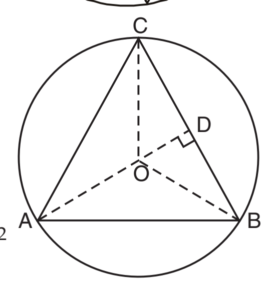
$\therefore \quad \angle ABO = \angle BOC = \angle CDA = 120°$

Draw $OD \perp BC$

Now in right ΔBDO,

$$\frac{BD}{BO} = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad BD = BO \times \frac{\sqrt{3}}{2} = 70 \times \sqrt{3} = \mathbf{35\sqrt{3}}\text{ cm}$$



Also $\quad BC = 2 \times BD = \mathbf{2 \times 35\sqrt{2}} = 70\sqrt{3}$ cm

$\therefore$ Area of equilateral $\Delta ABC = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times (70 \times \sqrt{3})^2 = \frac{\sqrt{3}}{4} \times 70 \times 70 \times 3\text{ cm}^2$$

$$= \sqrt{3} \times 3 \times 35 \times 35\text{ cm}^2 = 3675\sqrt{3}\text{ cm}^2$$

Also, area of the circle = πr^2

$$= \frac{22}{7} \times 70 \times 70\text{ cm}^2 = 22 \times 10 \times 70\text{ cm}^2 = 15400\text{ cm}^2$$

$\therefore$ Area of the shaded region

$$= 15400\text{ cm}^2 - (3675\sqrt{3})\text{ cm}^2 = 15400\text{ cm}^2 - (3675 \times 1.73)\text{ cm}^2$$

$$= 15400\text{ cm}^2 - 6357.75\text{ cm}^2 = \mathbf{9042.25\ cm^2.}$$

Q. 2. *Calculate the area other than the area common between two quadrants of the circles of radius 16 cm each, which is shown as the shaded region in the figure.* (AI CBSE 2009 C)

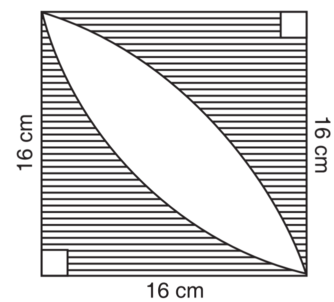
Sol. Area of sector *ADB*

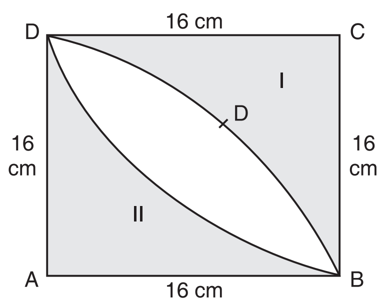
$$= \frac{90}{360} \times \frac{22}{7} \times 16 \times 16 \text{ cm}^2 \qquad \left[\text{using area} = \frac{\theta}{360}\pi r^2\right]$$

$$= \frac{1}{4} \times \frac{22}{7} \times 16 \times 16 \text{ cm}^2$$

$$= \frac{22 \times 4 \times 16}{7} \text{ cm}^2$$

$$= \frac{1408}{7} \text{ cm}^2$$





$\therefore$ Area of the shaded region-I $= \begin{bmatrix}\text{Area of the} \\ \text{square } ABCD\end{bmatrix} - \begin{bmatrix}\text{Area of the} \\ \text{sector } ADB\end{bmatrix}$

$$= 256 \text{ cm}^2 - \frac{1408}{7} \text{cm}^2 = \frac{1792 - 1708}{7} \text{cm}^2 = \frac{384}{7} \text{cm}^2$$

Similarly, in area of the shaded region-II $= \frac{384}{7} \text{cm}^2$

$\therefore$ Total area of the shaded region

= [Area of shaded region-I] + [Area of shaded region-II]

$$= \frac{384}{7} \text{cm}^2 + \frac{384}{7} \text{cm}^2 = \mathbf{\frac{768}{7} \text{cm}^2}.$$

Q. 3. *In the figure, PQ = 24 cm, PR = 7 cm and O is the centre of the circle. Find the area of shaded region. (Take* $\pi = 3.14$*)* (CBSE 2009)

Sol. In right Δ *RPQ*,

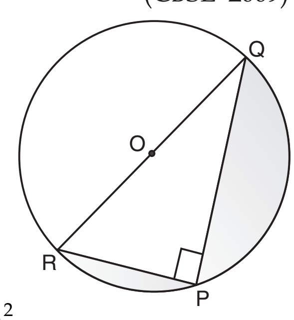
$$PR^2 + PQ^2 = RQ^2$$

$\Rightarrow \quad 7^2 + 24^2 = RQ^2$

$\Rightarrow \quad 49 + 576 = RQ^2$

$\Rightarrow \quad 625 = RQ^2 \Rightarrow RQ = \sqrt{625} = 25 \text{ cm}$

$\Rightarrow$ Radius of semi-circle $= \frac{25}{2}$ cm



$\therefore$ Area of semi-circle *RQP* $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{314}{100} \times \frac{25}{2} \times \frac{25}{2} \text{cm}^2$

$$= \frac{157}{4} \times \frac{25}{4} \text{cm}^2 = \frac{3925}{16} \text{cm}^2 = 245.31 \text{ cm}^2$$

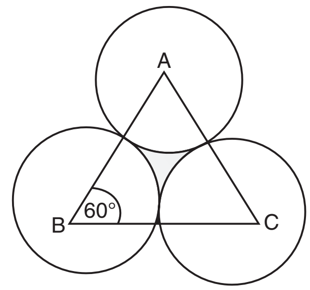
Area of right Δ *RPQ* $= \frac{1}{2} \times RP \times PQ = \frac{1}{2} \times 7 \times 24 \text{ cm}^2 = 7 \times 12 = 84 \text{ cm}^2$

$\therefore$ Area of the shaded region = Ar of semi-circle *RQP* – ar (right Δ *RPQ*)

$= 245.31 \text{ cm}^2 = 84 \text{ cm}^2 =$ **161.31 cm²**.

Q. 4. *The area of an equilateral triangle is $49\sqrt{3}\ cm^2$. Taking each angular point as centre, circles are drawn with radius equal to half length of the side of the triangle. Find the area of triangle not included in the circles.* *[Take $\sqrt{3}$ = 1.73]* (AI CBSE 2009)

Sol. Let the given equilateral triangle be *ABC*, such that its side = 14 cm.



$\therefore$ Area of Δ *ABC*

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (14)^2 \text{ cm}^2$$

$$= \frac{1.73}{4} \times 14 \times 14 \text{ cm}^2 = 49 \times 1.73 \text{ cm}^2$$

$$= 84.77 \text{ cm}^2$$

Since each angle of an equilateral triangle = 60°,

$\therefore$ Area of a sector having θ as 60° and radius $\frac{14}{2}$ *i.e.* 7 cm.

$$= \frac{60}{360} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{11 \times 7}{3} \text{ cm}^2 \qquad \left[\text{using area} = \frac{\theta}{360} \times \pi r^2\right]$$

$\Rightarrow$ Area of 3 sectors $= 3\left[\frac{11 \times 7}{3}\right] \text{cm}^2 = 77 \text{ cm}^2$

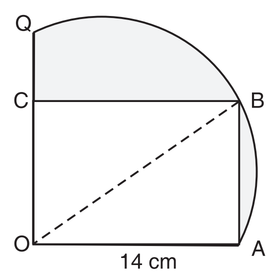
$\therefore$ Area of the shaded region

= [Area of equilateral Δ *ABC*] – [Area of 3 sectors]

$= 84.77 \text{ cm}^2 - 77 \text{ cm}^2 =$ **7.77 cm²**.

Q. 5. *A square OABC is inscribed in a quadrant OABQ of a circle as shown in the figure. If OA = 14 cm, find the area of the shaded region.* $\left[use\ \pi = \frac{22}{7}\right]$ (AI CBSE 2008 C, CBSE Delhi 2014)

Sol. *OABC* is a square with side = 14 cm.



$\therefore$ Area of the square *OABC* = 14 cm × 14 cm = 196 cm²

Now, the diagonal of the square *OABC*

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{14^2 + 14^2}$$

$$= 14\sqrt{2} \text{ cm}$$

$\Rightarrow$ Radius of the quadrant *OPBQ* $= 14\sqrt{2}$ cm

$\therefore$ Area of the quadrant *OABQ*

$$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times \left(14 \times \sqrt{2}\right)^2 \text{ cm}^2$$

$$= \frac{11}{7} \times \frac{14}{2} \times 14 \times 2 \text{ cm}^2 = \frac{11}{2} \times 2 \times 14 \times 2 \text{ cm}^2$$

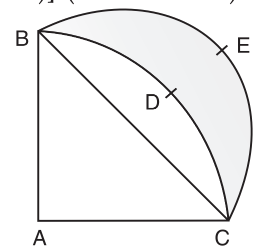
$$= 11 \times 14 \times 2 \text{ cm}^2 = 308 \text{ cm}^2$$

$\therefore$ Area of the shaded region

$= 308 \text{ cm}^2 - 196 \text{ cm}^2 =$ **112 cm²**.

Q. 6. *In the figure, ABDC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with diameter BC. Find the area of the shaded region.*[AI. CBSE (Foreign 2014)] (CBSE 2008 C)

Sol. We have, in the right Δ *ABC*,



$$BC^2 = AB^2 + AC^2 = 14^2 + 14^2 = 2\ (14)^2$$

$$\Rightarrow \quad BC = 14\sqrt{2}\ \text{cm}$$

∴ Radius of the semi-circle $= \frac{BC}{2} = \frac{14}{2}\sqrt{2} = 7\sqrt{2}$ cm

∴ Area of semi-circle *BEC*

$$= \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2 \text{cm}^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \times 2\ \text{cm}^2 = \mathbf{154\ cm^2}$$

Area of the quadrant with radius 14 cm,

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14\ \text{cm}^2 = \mathbf{154\ cm^2}$$

Area of right Δ *ABC* $= \frac{1}{2} \times 14 \times 14\ \text{cm}^2 = \mathbf{98\ cm^2}$

Area of the shaded region

$$= \begin{bmatrix}\text{Area of semicircle} \\ \text{with diameter } BC\end{bmatrix} + \begin{bmatrix}\text{Area of} \\ \Delta\ ABC\end{bmatrix} - \begin{bmatrix}\text{Area of} \\ \text{quadrant } ABDC\end{bmatrix}$$

$= 154\ \text{cm}^2 + 98\ \text{cm}^2 - 154\ \text{cm}^2 = \mathbf{98\ cm^2}$**.**

Q. 7. *In the figure, find the perimeter of the shaded region where, ADC, AEB and BFC are semi-circles on diameters AC, AB and BC respectively.* (CBSE 2008, CBSE Delhi 2014)

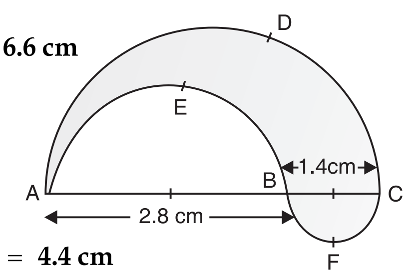
Sol. Diameter of semi-circle *ADC*

$= 2.8$ cm $+ 1.4$ cm $= 4.2$ cm

⇒ Radius of semi-circle *ADC* $= \frac{4.2}{2}$ cm $= 2.1$ cm

∴ Circumference of semi-circle *ADC*

$$= \frac{2}{2} \times \frac{22}{7} \times 2.1\ \text{cm} = \frac{2}{2} \times \frac{22}{7} \times \frac{21}{10} = \frac{66}{10}\ \text{cm} = \mathbf{6.6\ cm}$$



Diameter of semi-circle *AEB* = 2.8 cm

⇒ Radius of semi-circle *AEB* $= \frac{2.8}{2} = 1.4$ cm

∴ Circumference of semi-circle *AEB*

$$= \frac{2}{2} \times \pi \times r = \frac{2}{2} \times \frac{22}{7} \times 1.4\ \text{cm} = 22 \times \frac{2}{10}\ \text{cm} = \mathbf{4.4\ cm}$$

Diameter of semi-circle *BFC* = 1.4 cm

⇒ Radius of semi-circle *BFC* = 0.7 cm

∴ Circumference of semi-circle *BFC* $= \frac{2}{2} \times \frac{22}{7} \times 0.7 = \mathbf{2.2\ cm.}$

∴ Total perimeter of the shaded region = 6.6 cm + 4.4 cm + 2.2 cm = **13.2 cm.**

Q. 8. *In the figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. [use π = 3.14]* [NCERT Exemplar, CBSE 2012]

Sol. ∵ Δ*ABC* is an equilateral triangle.

$\therefore \quad \angle A = \angle B = \angle C = 60°$

Area of sector *AFEA*

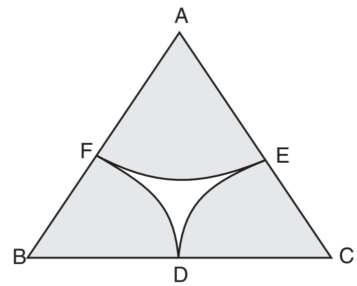
$$= \frac{\theta}{360} \times \pi r^2 \text{cm}^2$$

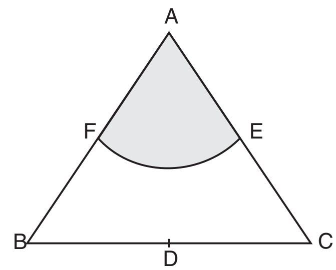
$$= \frac{60}{360} \times \pi (5)^2 \text{ cm}^2$$

$$\left[\because AF = \frac{1}{2}AB = \frac{1}{2}(10) \text{ cm} = 5 \text{ cm}\right]$$

$$= \frac{1}{6} \times 3.14 \times 5 \times 5 \text{ cm}^2$$

$$= \frac{78.5}{6} \text{ cm}^2$$





Area of all the three sectors = $3 \times \frac{78.5}{6}$ cm^2 = 39.25 cm^2.

Thus, area of the shaded region = 39.25 cm^2.

Q. 9. *In figure OABC is a quadrant of a circle of radius 7 cm. If OD = 4 cm, find the area of the shaded region. [Use* $\pi = \frac{22}{7}$] [CBSE (Foreign) 2014]

Sol. We have, the centre of the circle as 'O' and radius (r) = 7 cm

Area of the quadrant OABC $= \frac{1}{4}\pi r^2$

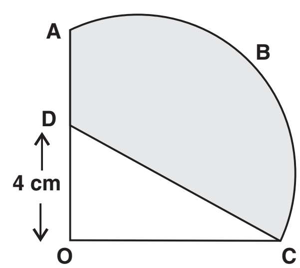
$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 \text{cm}^2 = \frac{11}{2} \times 7 \text{cm}^2 = \frac{77}{2} \text{cm}^2$$

Now, the ar(*rt* ΔCOD)

$$= \frac{1}{2} \times \text{OC} \times \text{OD}$$

$$= \frac{1}{2} \times 7 \times 4 \text{cm}^2 \qquad \left|\begin{array}{l}\because \text{OC} = \text{radius} = r = 7\text{cm} \\ \text{OD} = 4\text{cm (given)}\end{array}\right.$$

$$= 7 \times 2 = 14 \text{ cm}^2$$



$\therefore$ Area of the shaded region

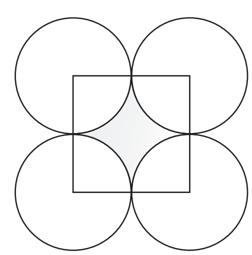
= (Area of the quadrant OABC) – (Area of ΔCOD)

$$= \left(\frac{77}{2} - 14\right) \text{cm}^2 = \frac{77 - 28}{2} \text{cm}^2 = \frac{49}{2} \text{cm}^2 = 24.5 \text{ cm}^2$$

TEST YOUR SKILLS

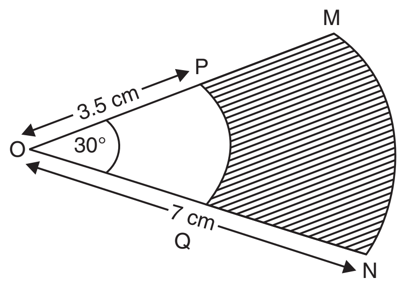
1. Four equal circles are described about the four corners of a square so that each touches two of the others, as shown in the figure. Find the area of the shaded region, each side of the square measuring 14 cm.

$\left(\text{Take } \pi = \frac{22}{7}\right)$



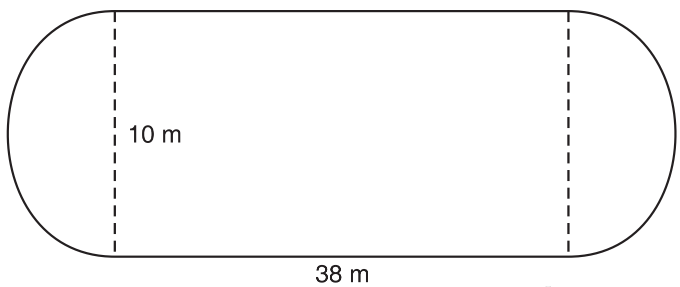
(CBSE 2012)

2. In the figure, *MN* and *PQ* are the arcs of two concentric circles of radii 7 cm and 3.5 cm respectively and $\angle MON = 30°$. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



(CBSE 2012)

3. A playground is in the form of a rectangle having semi-circles on the shorter sides as shown in the figure. Find its area when the length of the rectangular portion is 38 m and the breadth is 10 m. [use $\pi = 3.14$]

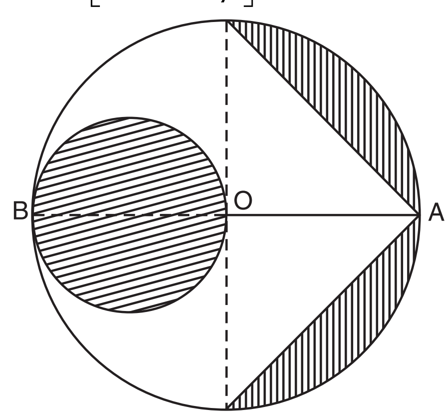


[NCERT Exemplar, CBSE 2006]

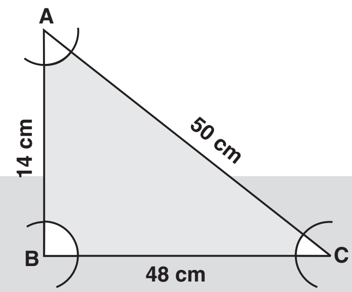
4. A chord of a circle of radius 20 cm subtends an angle of 90° at the centre. Find the area of the corresponding major segment of the circle.

[Üse $\pi = 3.14$] [NCERT Exemplar]

5. In the figure, *AB* is a diameter of the circle with centre *O* and *OA* = 7 cm. Find the area of the shaded region. $\left[\text{use } \pi = \frac{22}{7}\right]$ (CBSE 2012)



6. With the vertices A, B and C of a triangle ABC as centres arcs are drawn with radii 5 cm each as shown in the figure. If $AB = 14$ cm, $BC = 48$ cm and $CA = 50$ cm, then find the area of the shaded region. [Use $\pi = 3.14$] [NCERT Exemplar]



***Hint*:**

$a = 48$ cm, $b = 50$ cm and $c = 14$ cm

$$\Rightarrow s = \frac{48+50+14}{2} cm = 56 cm$$

Area of ΔABC

$$= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{56 \times 8 \times 6 \times 42}\ cm^2$$

$$= 336\ cm^2$$

$$\text{Area of three sectors} = \left[\pi r^2 \cdot \frac{\angle A}{360°} + \pi r^2 \cdot \frac{\angle B}{360°} + \pi r^2 \cdot \frac{\angle C}{360°}\right]$$

$$= \frac{\pi r^2}{360°}[\angle A + \angle B + \angle C] = \frac{\pi r^2}{360°} \times 180°$$

$$= 3.14 \times 5^2 \times \frac{180}{360} = \frac{314}{100} \times \frac{25}{2} = 39.25\ cm^2$$

$$\Rightarrow \text{Area of shaded region} = 336\ cm^2 - 39.25\ cm^2 = 295.75\ cm^2$$

7. Area of a sector of a circle of radius 36 cm is $54\pi\ cm^2$. Find the length of the corresponding arc of the sector. [NCERT Exemplar]

***Hint*:**

$$\text{Length of an arc} = \frac{\theta}{360°} \times 2\pi r$$

$$\text{Area of a sector} = \frac{\theta}{360°} \times \pi r^2$$

$$\Rightarrow \frac{\theta}{360°} \times \pi \times (36)^2 = 54\pi$$

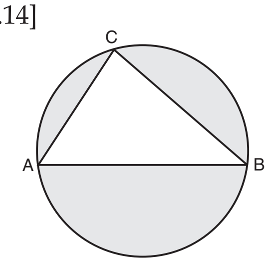
$$\Rightarrow \theta = \frac{54 \times 360}{36 \times 36} = 15°$$

$$\text{Now, length of the arc} = \frac{15}{360°} \times 2 \times \pi \times 36\ cm$$

$$= 3\pi\ cm$$

8. A bicycle wheel makes revolutions per minute to maintain a speed of 8.91 km per hour. Find the diameter of the wheel. (CBSE 2012)

9. In the figure, AB is a diameter of the circle, $AC = 6$ cm and $BC = 8$ cm. Find the area of the shaded region.[Use $\pi = 3.14$] [NCERT Exemplar]



Hint:

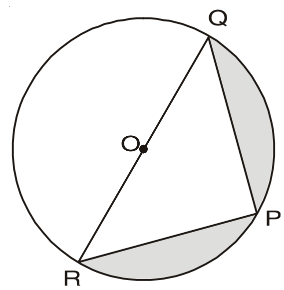
AB is diameter

$\Rightarrow \Delta ABC$ *is in a semicircle*

$\Rightarrow \quad \angle C = 90°$

$\therefore \quad AB^2 = AC^2 + BC^2$

10. Find the area of the shaded region in the given figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle. (CBSE 2012)



Hint:

$\because$ *O is the centre*

$\therefore$ *RQ is a diameter*

$\Rightarrow \Delta PQR$ *is a rt* Δ *being in a semi circle*

$$\therefore \quad RQ = \sqrt{RP^2 + PQ^2}$$

$\therefore$ *Area of the shaded region = [Area of semi circle –ar* ΔPQR*]*

11. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5 m, find the increase in area of the grassy lawn in which the calf can graze. [NCERT Exemplar]

Hint:

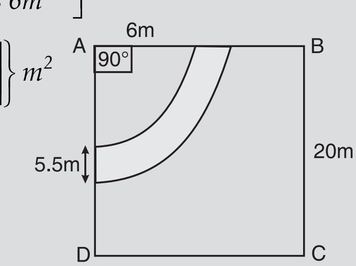
Increased radius = *6 m + 5.5 m = 11.5 m*

$$\text{Increase in area} = \begin{bmatrix}\text{Area of sector of} \\ \text{central angle } 90° \\ \text{and radius } 11.5 \text{ cm}\end{bmatrix} - \begin{bmatrix}\text{Area of sector of} \\ \text{central angle } 90° \\ \text{and radius } 6\text{m}\end{bmatrix}$$

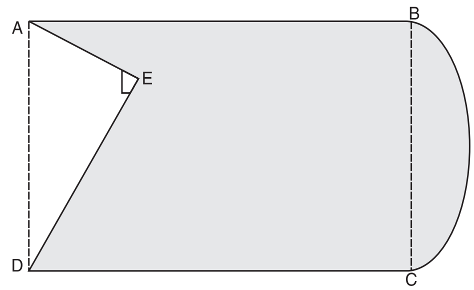
$$= \left\{\left[\frac{90}{360} \times \pi(11.5)^2\right] - \left[\frac{90}{360} \times \pi \times 6^2\right]\right\} m^2$$

$$= \frac{\pi}{4}\left[(11.5)^2 - (6)^2\right] m^2$$

$$= \frac{22}{7} \times \frac{1}{4} \times \left[(11.5 + 6)(11.5 - 6)\right] m^2$$



12. In the figure, from a rectangular region ABCD with AB = 20 cm, a right triangle AED with AE = 9 cm and DE = 12 cm, is cut off. On the other end, taking BC as diameter, a semi-circle is added on outside the region. Find the area of the shaded region. [Use = $\pi = 3.14$] [AI. CBSE Foreign 2014]



***Hint*:**

$$AD = \sqrt{DE^2 + AE^2} = \sqrt{12^2 + 9^2} = 15cm$$

ar (ABCD) $= AB \times AD = 20 \times 15\ cm^2 = 300\ cm^2$

ar (rt ΔAED) $= \frac{1}{2} \times AE \times DE = \frac{1}{2} \times 9 \times 12 = 88.31cm^2$

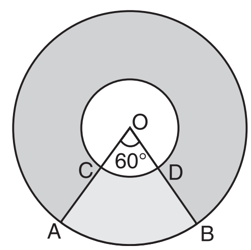
ar (semi circle with diameter BC) $= \frac{1}{2}(\pi r^2) = \frac{1}{2} \times 3.14 \times (7.5)^2 = 88.31cm^2$

Area of shaded region

= ar.(ABCD) + ar. (Semi circle) – ar.(rt Δ)

$= 300cm^2 + 88.31cm^2 - 54cm^2 = \mathbf{334.31cm^2}$

13. In the figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]. [AI. CBSE 2014]

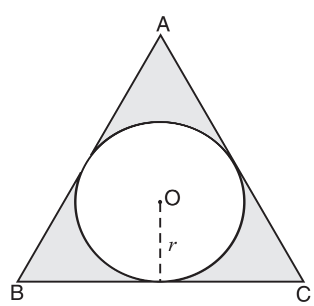


***Hint*:**

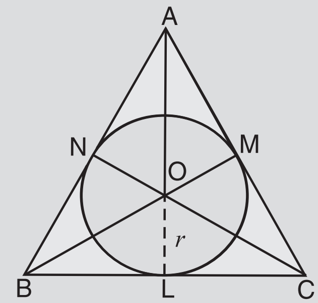
Area of shaded region = [Area of larger circle – area of smaller circle – sector AOB + sector COD]

$$\frac{22}{7}\left[42^2 - 21^2\right] - \frac{1}{6} \times \frac{22}{7}(42)^2 + \frac{1}{6} \times \frac{22}{7}(21)^2 = 5544 - 1386 - 924 + 231 = 3465cm^2$$

14. In the figure, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of the inscribed circle and the area of the shaded region. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]



***Hint*:**



Area of equilatral $\Delta ABC = \frac{\sqrt{3}}{4}(side)^2 = \frac{\sqrt{3}}{4} \times 12 \times 12 cm^2$

$= 1.73 \times 3 \times 12 cm^2 = 62.28 cm^2$

Area of $\Delta BOC = \frac{1}{3} ar(\Delta ABC) = \frac{1}{3} \times 62.28 = 20.76 cm^2$

$\therefore \quad \frac{1}{2} \times BC \times r = 20.76 \Rightarrow r = 3.46 cm$

$\Rightarrow$ *Area of the circle with radius 3.46 cm*

$= \pi \times r^2 = 3.14 \times 3.46 \times 3.46 \ cm^2$

$= 37.6 \ cm^2$

Now, area of shaded region = (ar Equilateral ΔABC*)*

– (Circle with radius 3.46 cm)

$= 62.28 \ cm^2 - 37.6 \ cm^2 = 24.68 cm^2$

ANSWERS

Test Your Skills

1. 12 cm^2	**2.** 9.625 cm^2	**3.** (380 + 25 π) m^2
4. 285.5 cm^2	**5.** 66.5 cm^2	**6.** 295.75 cm^2
7. 3 π cm	**8.** 0.63 m	**9.** 54.5 cm^2
10. 161.54 cm^2	**11.** 75.625 m^2	

❑❑❑