

QUADRATIC EQUATION

✓ (A) OBJECTIVE TYPE QUESTIONS

1 Mark Each



Stand Alone MCQs

(1 Mark Each)

1. Which of the following is a quadratic equation ?

- (A) $x^2 + 2x + 1 = (4 - x)^2 + 3$
 (B) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$
 (C) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k = -1$
 (D) $x^3 - x^2 = (x - 1)^3$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned} x^3 - x^2 &= x^3 - 1 - 3x(x - 1) \\ x^3 - x^2 &= x^3 - 1 - 3x^2 + 3x \\ -x^2 + 3x^2 + 1 - 3x &= 0 \\ 2x^2 - 3x + 1 &= 0 \end{aligned}$$

It is of the form of $ax^2 + bx + c = 0$.

2. Which of the following is not a quadratic equation?

- (A) $2(x - 1)^2 = 4x^2 - 2x + 1$
 (B) $2x - x^2 = x^2 + 5$
 (C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$
 (D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} (\sqrt{2}x)^2 + (\sqrt{3})^2 + 2 \times \sqrt{2}x \times \sqrt{3} + x^2 &= 3x^2 - 5x \\ 2x^2 + 3 + 2\sqrt{6}x + x^2 &= 3x^2 - 5x \\ 3x^2 + 2\sqrt{6}x + 3 &= 3x^2 - 5x \\ x(5 + 2\sqrt{6}) + 3 &= 0 \end{aligned}$$

It is not of the form of $ax^2 + bx + c = 0$.

3. Which of the following equations has 2 as its root ?

- (A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$
 (C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

Ans. Option (C) is correct.

Explanation:

Put the value of $x = 2$ in $2x^2 - 7x + 6 = 0$

$$\begin{aligned} 2(2)^2 - 7(2) + 6 &= 0 \\ 8 - 14 + 6 &= 0 \\ 0 &= 0 \end{aligned}$$

So, $x = 2$ is a root of $2x^2 - 7x + 6 = 0$.

Q. 4. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then

the value of k is:

- (A) 2 (B) -2
 (C) $\frac{1}{2}$ (D) $\frac{1}{2}$

Ans. Option (A) is correct.

Explanation: Since, $\frac{1}{2}$ is a root of the equation

$$\begin{aligned} x^2 + kx - \frac{5}{4} &= 0, \\ \text{Then, } \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} &= 0 \\ \frac{1}{4} + \frac{k}{2} - \frac{5}{4} &= 0 \\ \frac{k}{2} &= \frac{5}{4} - \frac{1}{4} \\ \frac{k}{2} &= 1 \\ k &= 2. \end{aligned}$$

5. Which of the following equations has the sum of its roots as 3 ?

- (A) $2x^2 - 3x + 6 = 0$
 (B) $-x^2 + 3x - 3 = 0$

(C) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$

(D) $3x^2 - 3x + 3 = 0$

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Ans. Option (B) is correct.

Explanation: $-x^2 + 3x - 3 = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = -1, b = 3, c = -3$$

$$\Rightarrow \text{Sum of the roots} = \frac{-b}{a} = \frac{-3}{-1} = 3$$

6. The roots of the equation $x^2 + 7x + 10 = 0$ are:

(A) $-5, -2$

(B) $5, 2$

(C) $5, -2$

(D) $-5, 2$

U [CBSE SQP, 2020-21]

Ans. Option (A) is correct.

Explanation: Given, $x^2 + 7x + 10 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 7 \text{ and } c = 10$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{(7)^2 - 4 \times 1 \times 10}}{2 \times 1}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{9}}{2}$$

$$\Rightarrow x = \frac{-7 \pm 3}{2}$$

$$\Rightarrow x = \frac{-7+3}{2} \text{ or } \frac{-7-3}{2}$$

$$\Rightarrow x = -2 \text{ or } -5$$

Hence, the roots of the given equation are -2 and -5 .

7. Which of the following equations has two distinct real roots?

(A) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

(B) $x^2 + x - 5 = 0$

(C) $x^2 + 3x + 2\sqrt{2} = 0$

(D) $5x^2 - 3x + 1 = 0$

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Ans. Option (B) is correct.

Explanation: $x^2 + x - 5 = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 1, b = 1, c = -5$$

$$b^2 - 4ac = (1)^2 - 4(1)(-5)$$

$$= 1 + 20$$

$$= 21 > 0$$

Since D (i.e., $b^2 - 4ac$) > 0

Hence, the equation has two distinct real roots.

8. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots, is:

(A) 0 only

(B) 4

(C) 8 only

(D) 0, 8

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Ans. Option (D) is correct.

Explanation: Given equation is $2x^2 - kx + k = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 2, b = -k, c = k$$

For equal roots $b^2 - 4ac = 0$

$$(-k)^2 - 4(2)(k) = 0$$

$$k^2 - 8k = 0$$

$$k(k - 8) = 0$$

$$k = 0, 8$$

Hence, the required values of k are 0 and 8.

9. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) two distinct real roots

(B) two equal real roots

(C) no real root

(D) more than 2 real roots

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Ans. Option (C) is correct.

Explanation: $2x^2 - \sqrt{5}x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 2, b = -\sqrt{5}, c = 1$$

$$\text{Discriminant} = b^2 - 4ac = (-\sqrt{5})^2 - 4(2)(1) \\ = 5 - 8 = -3 < 0$$

Since D (i.e., $b^2 - 4ac$) < 0

Therefore, the equation has no real root.

10. Which of the following equations has no real root?

(A) $x^2 - 4x + 3\sqrt{2} = 0$

(B) $x^2 + 4x - 3\sqrt{2} = 0$

(C) $x^2 - 4x - 3\sqrt{2} = 0$

(D) $3x^2 + 4\sqrt{3}x + 4 = 0$

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Ans. Option (A) is correct.

Explanation: $x^2 - 4x + 3\sqrt{2} = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 1, b = -4, c = 3\sqrt{2}$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (-4)^2 - 4(1)(3\sqrt{2})$$

$$= 16 - 12\sqrt{2}$$

$$= 16 - 12 \times 1.41$$

$$= 16 - 16.92$$

$$= -0.92 < 0$$

Since, $D < 0$

Therefore, the equation has no real roots.

11. $(x^2 + 1)^2 - x^2 = 0$ has

(A) four real roots

(B) two real roots

(C) no real roots

(D) one real root.

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Ans. Option (C) is correct.

Explanation: $(x^2 + 1)^2 - x^2 = 0$

$$x^4 + 1 + 2x^2 - x^2 = 0$$

$$x^4 + x^2 + 1 = 0$$

Let $x^2 = y$

$$y^2 + y + 1 = 0$$

On comparing with $ay^2 + by + c = 0$

$$a = 1, b = 1, c = 1$$

$$\text{Discriminant, } D = b^2 - 4ac$$

$$= (1)^2 - 4(1)(1)$$

$$= 1 - 4$$

$$= -3 < 0$$

Since $D < 0$

Therefore, the equation has no real roots.



Case-based MCQs

(1 Mark Each)

I. Read the following text and answer the following questions on the basis of the same:

Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km.

[CBSE QB, 2021]



1. What will be the distance covered by Ajay's car in two hours ?

- (A) $2(x + 5)$ km (B) $(x - 5)$ km
 (C) $2(x + 10)$ km (D) $(2x + 5)$ km

Ans. Option (A) is correct.

Explanation:

Speed of Raj's car = x km/hr
 Speed of Ajay's car = $(x + 5)$ km/hr
 Distance covered by Ajay in 2 hours
 $= [(x + 5) \times 2]$ km
 $= 2(x + 5)$ km.

2. Which of the following quadratic equation describe the speed of Raj's car ?

- (A) $x^2 - 5x - 500 = 0$ (B) $x^2 + 4x - 400 = 0$
 (C) $x^2 + 5x - 500 = 0$ (D) $x^2 - 4x + 400 = 0$

Ans. Option (C) is correct.

Explanation: Given,

Speed of Raj's car = x km/hour
 Speed of Ajay's car = $x + 5$ km/hour
 Time taken by Raj = $\frac{400}{x}$ hours
 Time taken by Ajay = $\frac{400}{(x + 5)}$ hours

Raj took 4 hours more than Ajay i.e.,

$$\frac{400}{x} - \frac{400}{x + 5} = 4$$

$$\Rightarrow \frac{100}{x} - \frac{100}{x + 5} = 1$$

$$\Rightarrow 100(x + 5) - 100x = x(x + 5)$$

$$\Rightarrow x^2 + 5x - 500 = 0$$

3. What is the speed of Raj's car ?

- (A) 20 km/hour (B) 15 km/hour
 (C) 25 km/hour (D) 10 km/hour

Ans. Option (A) is correct.

Explanation: Solving quadratic equation

$$x^2 + 5x - 500 = 0,$$

we get

$$(x + 25)(x - 20) = 0$$

$$\Rightarrow x = -25 \text{ or } x = 20$$

$$\Rightarrow x = 20$$

(as x can't be negative)

So, speed of Raj's car is 20 km/hour

4. How much time took Ajay to travel 400 km ?

- (A) 20 h (B) 40 h
 (C) 25 h (D) 16 h

Ans. Option (D) is correct.

Explanation: Time taken by Ajay = $\frac{400}{x + 5}$ h

$$= \frac{400}{20 + 5} \text{ h}$$

$$= 16 \text{ h}$$

5. The Speed of Ajay's car is:

- (A) 25 km/hour (B) 50 km/hour
 (C) 75 km/hour (D) None of these

Ans. Option (A) is correct.

Explanation: Speed of Ajay's car

$$= x + 5 \text{ km/hour}$$

$$= 20 + 5$$

$$= 25 \text{ km/hour}$$

II. Read the following text and answer the following questions on the basis of the same:

The speed of a motor boat is 20 km/hr., For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.

[CBSE QB, 2021]



Downstream (a)

Upstream (b)

1. Let speed of the stream be x km/hr. then speed of the motorboat in upstream will be

- (A) 20 km/hr (B) $(20 + x)$ km/hr
 (C) $(20 - x)$ km/hr (D) 2 km/hr

Ans. Option (C) is correct.

Explanation: Speed of motorboat in upstream
 = Speed of motorboat
 – Speed of stream
 = $(20 - x)$ km/hr

2. What is the relation between speed, distance and time ?

- (A) speed = (distance)/time
 (B) distance = (speed)/time
 (C) time = speed \times distance
 (D) speed = distance \times time

Ans. Option (A) is correct.

3. Which is the correct quadratic equation for the speed of the current ?

- (A) $x^2 + 30x - 200 = 0$ (B) $x^2 + 20x - 400 = 0$
 (C) $x^2 + 30x - 400 = 0$ (D) $x^2 - 20x - 400 = 0$

Ans. Option (C) is correct.

Explanation: In downstream:

$$\begin{aligned}\text{Speed} &= (20 + x) \text{ km/hour} \\ \text{Distance} &= 15 \text{ km} \\ \text{Time} &= \frac{15}{20 + x}\end{aligned}$$

In upstream:

$$\begin{aligned}\text{Speed} &= (20 - x) \text{ km/hour} \\ \text{Distance} &= 15 \text{ km} \\ \text{Time} &= \frac{15}{20 - x}\end{aligned}$$

According to question, for covering the distance of 15 km the boat took 1 hour more for upstream than downstream.

So,

$$\begin{aligned}\frac{15}{20 - x} &= \frac{15}{20 + x} + 1 \\ \Rightarrow \frac{15}{20 - x} - \frac{15}{20 + x} &= 1 \\ \frac{15(20 + x) - 15(20 - x)}{(20 - x)(20 + x)} &= 1 \\ \Rightarrow 30x &= 400 - x^2 \\ \Rightarrow x^2 + 30x - 400 &= 0\end{aligned}$$

4. What is the speed of current ?

- (A) 20 km/hour (B) 10 km/hour
 (C) 15 km/hour (D) 25 km/hour

Ans. Option (B) is correct.

Explanation: On solving the quadratic equation

$$\begin{aligned}x^2 + 30x - 400 &= 0, \text{ we get} \\ (x - 10)(x + 40) &= 0 \\ \Rightarrow x &= 10 \text{ or } x = -40 \\ \text{So, Speed of the stream} &= 10 \text{ km/hour}\end{aligned}$$

5. How much time boat took in downstream?

- (A) 90 minute (B) 15 minute
 (C) 30 minute (D) 45 minute

Ans. Option (C) is correct.

Explanation: Time taken in downstream

$$\begin{aligned}&= \frac{15}{20 + x} \\ &= \frac{15}{20 + 10} \\ &= \frac{1}{2} \text{ hours} \\ &= 30 \text{ minutes}\end{aligned}$$

AI III. Read the following text and answer the following questions on the basis of the same:

John and Jivanti are playing with the marbles. They together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. C



1. If John had x number of marbles, then number of marbles Jivanti had:

- (A) $x - 45$ (B) $45 - x$
 (C) $45x$ (D) $x - 5$

Ans. Option (B) is correct.

Explanation: If John had x number of marbles, then Jivanti had $(45 - x)$ marbles, because there are total 45 marbles.

2. Number of marbles left with Jivanti, when she lost 5 marbles:

- (A) $x - 45$ (B) $40 - x$
 (C) $45 - x$ (D) $x - 40$

Ans. Option (B) is correct.

Explanation: Number of marbles left with Jivanti, when she lost 5 marbles

$$\begin{aligned}&= (45 - x - 5) \\ &= (40 - x)\end{aligned}$$

3. The quadratic equation related to the given problem is:

- (A) $x^2 - 45x + 324 = 0$ (B) $x^2 + 45x + 324 = 0$
 (C) $x^2 - 45x - 324 = 0$ (D) $-x^2 - 45x + 324 = 0$

Ans. Option (A) is correct.

Explanation: According to question,

$$\begin{aligned}(x - 5)(40 - x) &= 124 \\ \Rightarrow -x^2 - 200 + 40x + 5x - 124 &= 0 \\ \Rightarrow x^2 - 45x + 324 &= 0\end{aligned}$$

4. Number of marbles John had:

- (A) 10 (B) 9
 (C) 35 (D) 30

Ans. Option (B) is correct.

Explanation:

$$x^2 - 45x + 324 = 0$$

$$\Rightarrow x^2 - 9x - 36x + 324 = 0$$

$$\Rightarrow x(x - 9) - 36(x - 9) = 0$$

$$\Rightarrow (x - 9)(x - 36) = 0$$

Either $x = 9$ or $x = 36$.

Therefore, the number of marbles John had 9 or 36.

5. If John had 36 marbles, then number of marbles Jivanti had:

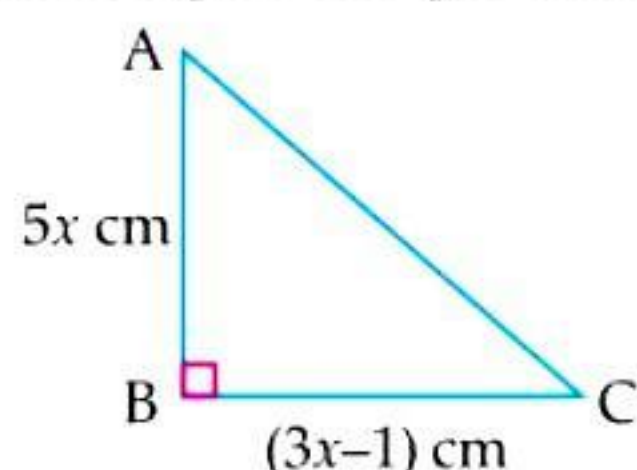
- (A) 10 (B) 9
(C) 36 (D) 35

Ans. Option (B) is correct.

Explanation: If John had 36 marbles, then Jivanti had $(45 - 36) = 9$ marbles.

AI IV. Read the following text and answer the following questions on the basis of the same:

There is a triangular playground as shown in the figure below. Many Children and people are playing and walking in the ground.



As we see in the above figure of right angled triangle playground, the length of the sides are $5x$ cm and $(3x - 1)$ cm and area of the triangle is 60 cm^2 . C + AE

1. The value of x is:

- (A) 8 (B) 3
(C) 4 (D) 5

Ans. Option (B) is correct.

Explanation: Given, area of triangle = 60 cm^2

$$\Rightarrow \frac{1}{2} \times AB \times BC = 60$$

$$\Rightarrow (5x)(3x - 1) = 120$$

$$\Rightarrow 3x^2 - x - 24 = 0$$

$$\Rightarrow 3x^2 - 9x + 8x - 24 = 0$$

$$\Rightarrow 3x(x - 3) + 8(x - 3) = 0$$

$$\Rightarrow (x - 3)(3x + 8) = 0$$

Either $x = 3$ or $x = -\frac{8}{3}$

Since length can't be negative, then $x = 3$.

2. The length of AB is:

- (A) 8 cm (B) 10 cm
(C) 15 cm (D) 17 cm

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \text{The length of AB} &= 5x \text{ cm} \\ &= 5 \times 3 \text{ cm} \\ &= 15 \text{ cm} \end{aligned}$$

3. The length of AC is:

- (A) 17 cm (B) 15 cm
(C) 21 cm (D) 20 cm

Ans. Option (A) is correct.

Explanation: $\because AB = 15 \text{ cm}$ and $BC = (3x - 1) \text{ cm}$
 $= (3 \times 3 - 1) \text{ cm}$
 $= 8 \text{ cm}$

Now, in right angled $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$

(By using Pythagoras theorem)

$$= (15)^2 + (8)^2$$

$$= 225 + 64$$

$$= 289 = (17)^2$$

Hence, $AC = 17 \text{ cm}$.

4. The perimeter of $\triangle ABC$ is:

- (A) 35 cm (B) 45 cm
(C) 30 cm (D) 40 cm

Ans. Option (D) is correct.

Explanation: Here, $AB = 15 \text{ cm}$, $BC = 8 \text{ cm}$ and $AC = 17 \text{ cm}$.

Then, the perimeter of $\triangle ABC$

$$= (AB + BC + CA) \text{ cm}$$

$$= (15 + 8 + 17) \text{ cm}$$

$$= 40 \text{ cm}.$$

5. The given problem is based on which mathematical concept?

- (A) AP
(B) Linear equation in one variable
(C) Quadratic Equations
(D) None of these

Ans. Option (C) is correct.

Explanation: The given problem is based on the concept of quadratic equations.

(B) SUBJECTIVE QUESTIONS

Very Short Answer Type Questions (1 Mark Each)

AI 1. Find the roots of the equation $x^2 + 7x + 10 = 0$.

U [CBSE SQP, 2020-21]

Sol. $x^2 + 7x + 10 = 0$

$$x^2 + 5x + 2x + 10 = 0$$

$$(x + 5)(x + 2) = 0$$

Either $x = -5$ and $x = -2$

[CBSE SQP Marking Scheme, 2020-21]

Detailed Solution:

Given, $x^2 + 7x + 10 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = 7$ and $c = 10$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{(7)^2 - 4 \times 1 \times 10}}{2 \times 1}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{9}}{2}$$

$$\Rightarrow x = \frac{-7 \pm 3}{2}$$

$$\Rightarrow x = \frac{-7+3}{2} \text{ or } \frac{-7-3}{2}$$

$$\Rightarrow x = -2 \text{ or } -5$$

Hence, the roots of the given equation are -2 and -5 .

[AI] 2. Find the roots of the quadratic equation $x^2 - 0.04 = 0$. **[A]** [CBSE OD Set-I, 2020]

Sol. $x^2 - 0.04 = 0$

$$\Rightarrow x^2 = 0.04$$

$$\Rightarrow x = \pm \sqrt{0.04}$$

$$\Rightarrow x = \pm 0.2. \quad 1$$

[CBSE SQP Marking Scheme, 2020]

3. If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other, then find the value of k .

[R] + [U] [CBSE SQP, 2020]

Sol. 4. 1

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Let one root $= \alpha$

and the other root $= \frac{1}{\alpha}$

$$\text{Product of roots} = \alpha \times \frac{1}{\alpha}$$

Given equation is

$$(k-1)x^2 - 10x + 3 = 0$$

Comparing it with $ax^2 + bx + c = 0$, we get $a = (k-1)$, $b = 10$ and $c = 3$

$$\therefore \text{Product of roots} = \frac{c}{a}$$

$$\therefore \frac{3}{k-1} = \alpha \times \frac{1}{\alpha}$$

$$\text{or, } \frac{3}{k-1} = 1$$

$$\text{or, } 3 = k-1$$

$$\text{or, } k = 4.$$

4. Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other. **[A]** [CBSE Delhi Set-I, II, III, 2019]

Sol. Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$$\Rightarrow \text{Product of roots} = 1 \quad \frac{1}{2}$$

$$\Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3 \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given equation: $3x^2 - 10x + k = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -10, c = k$$

Let one root be α so other root is $\frac{1}{\alpha}$

$$\text{Now, product of roots} = \frac{c}{a}$$

$$\alpha \times \frac{1}{\alpha} = \frac{k}{3}$$

$$\therefore k = 3$$

Hence, value of k is 3.

5. Write the discriminant of the quadratic equation $(x+5)^2 = 2(5x-3)$.

[A] [CBSE, 2019]

**Topper Answer, 2019**

Sol. $(x+5)^2 = 2(5x-3)$

$$\Rightarrow x^2 + 25 + 10x = 10x - 6$$

$$\Rightarrow x^2 + 31 = 0$$

$$a = 1, b = 0, c = 31$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= 0^2 - 4 \times 1 \times 31$$

$$= 0 - 124$$

$$= -124$$

6. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k .

[C] + [U] [CBSE Delhi & OD 2018]

Sol. $x = 3$ is one root of the equation

$$\therefore 9 - 6k - 6 = 0$$

$\frac{1}{2}$

$$\Rightarrow k = \frac{1}{2}$$

[CBSE Marking Scheme, 2018] $\frac{1}{2}$



Topper Answer, 2019

$x^2 - 2kx - 6 = 0$. let α be other root.
 Product = $\frac{c}{a} = \frac{-6}{1} = -6$.
 $3 \times \alpha = -6$
 $\alpha = -2$.
 Sum = $-\frac{b}{a} = -\frac{(-2k)}{1} = 2k$.
 $\therefore 3 + (-2) = 2k$
 $1 = 2k, k = \frac{1}{2}$
 Value of k is $\frac{1}{2}$

7. Find the value of k , for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is 2.

[U] [CBSE SQP, 2018]

Sol. Since, $x = 2$ is one root of the equation.

$$4k - 28 + 8 = 0$$

$$4k = 20$$

$$k = 5$$

8. Find the positive root of $\sqrt{3x^2 + 6} = 9$.

[U] [CBSE Term-II 2015, 2017]

Sol. $\sqrt{3x^2 + 6} = 9$

$$3x^2 + 6 = 81$$

$$3x^2 = 81 - 6 = 75$$

$$x^2 = \frac{75}{3} = 25$$

$$\therefore x = \pm 5$$

Hence, positive root = 5.

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[CBSE Marking Scheme, 2015]

[AI] 9. If $x = -\frac{1}{2}$, is a solution of the quadratic equation

$$3x^2 + 2kx - 3 = 0, \text{ find the value of } k. \quad [C] + [U]$$

[CBSE Term-II, 2015 & CBSE Delhi Set-I, II, III, 2017]

Sol. Putting $x = -\frac{1}{2}$ in $3x^2 + 2kx - 3 = 0$

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\Rightarrow k = \frac{3}{4} - 3$$

$$\Rightarrow k = \frac{3-12}{4}$$

Hence,

$$k = \frac{-9}{4}$$

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[CBSE Marking Scheme, 2015]

10. Find the roots of the quadratic equation

$$\sqrt{3}x^2 - 2x - \sqrt{3} = 0. \quad [U] \text{ [CBSE Term-II 2015]}$$

Sol. Given, $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$

$$\Rightarrow \sqrt{3}x^2 - 3x + x - \sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}x(x - \sqrt{3}) + 1(x - \sqrt{3}) = 0$$

$$\Rightarrow (x - \sqrt{3})(\sqrt{3}x + 1) = 0$$

$$\therefore x = \sqrt{3} \text{ or } \frac{-1}{\sqrt{3}}$$

[AI] 11. For what values of k , the given quadratic equation $9x^2 + 6kx + 4 = 0$ has equal roots?

[C] + [A] [CBSE SQP, 2020-21]

Sol. $9x^2 + 6kx + 4 = 0$

For equal roots $b^2 - 4ac = 0$

Discriminant, $D = 0$

so, $(6k)^2 - 4 \times 9 \times 4 = 0$

$\frac{1}{2}$

$$36k^2 = 144$$

$$k^2 = 4$$

$$k = \pm 2$$

$\frac{1}{2}$

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

Given, $9x^2 + 6kx + 4 = 0$.

Comparing with $ax^2 + bx + c = 0$,

we get $a = 9, b = 6k, c = 4$

Since, Discriminant, $D = b^2 - 4ac$

and for equal roots,

$$\begin{aligned}
 b^2 - 4ac &= 0 & [\because D = 0] \\
 \Rightarrow (6k)^2 - 4 \times 9 \times 4 &= 0 \\
 \Rightarrow 36k^2 - 144 &= 0 \\
 \Rightarrow 36k^2 &= 144 \\
 \Rightarrow k^2 &= 4 \\
 \Rightarrow k &= \pm 2.
 \end{aligned}$$

[AI] 12. For what value(s) of 'a' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots?

[A] [CBSE SQP, 2020-21]

Sol. Given that,

$$\begin{aligned}
 3ax^2 - 6x + 1 &= 0 \\
 \text{For no real roots, } b^2 - 4ac &< 0 & \frac{1}{2} \\
 \text{Discriminant, } D &< 0 \\
 \text{so, } (-6)^2 - 4(3a)(1) &< 0 \\
 12a &> 36 \\
 a &> 3 & \frac{1}{2}
 \end{aligned}$$

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

$$\begin{aligned}
 \text{Given, } 3ax^2 - 6x + 1 &= 0 \\
 \text{On Comparing with } ax^2 + bx + c &= 0, \\
 \text{we get } a = 3a, b = -6 \text{ and } c = 1 \\
 \text{Discriminant, } D = b^2 - 4ac \\
 &= (-6)^2 - 4 \times 3a \times 1 \\
 &= 36 - 12a \\
 \text{For condition of 'no real roots',} \\
 b^2 - 4ac &< 0 \\
 \Rightarrow 36 - 12a &< 0 \\
 \Rightarrow 12a &> 36 \\
 \Rightarrow a &> 3.
 \end{aligned}$$

[AI] 13. Find the values(s) of k for which the quadratic equation $x^2 + 2\sqrt{2}kx + 18 = 0$ has equal roots.

[C] + [A] [CBSE SQP, 2020]

$$\begin{aligned}
 \text{Sol. } D &= (2\sqrt{2}k)^2 - 4(1)(18) = 0 & \frac{1}{2} \\
 \Rightarrow k &= \pm 3 & \frac{1}{2} \\
 & \text{[CBSE Marking Scheme, 2020]}
 \end{aligned}$$

Detailed Solution:

Since, $x^2 + 2\sqrt{2}kx + 18 = 0$ has equal roots

$$\begin{aligned}
 D &= 0 \\
 b^2 &= 4ac \\
 (2\sqrt{2}k)^2 &= 4 \times 1 \times 18 \\
 \Rightarrow 4 \times 2 \times k^2 &= 72 \\
 \Rightarrow k^2 &= \frac{72}{8} \\
 \Rightarrow k^2 &= 9 \\
 \Rightarrow k &= \sqrt{9} \\
 \Rightarrow k &= \pm 3.
 \end{aligned}$$

14. For what values of k , the roots of the equation $x^2 + 4x + k = 0$ are real?

[A] [CBSE Delhi Set-I, II, III, 2019]

$$\begin{aligned}
 \text{Sol. Since roots of the equation } x^2 + 4x + k &= 0 \text{ are real} \\
 \Rightarrow 16 - 4k &\geq 0 & \frac{1}{2} \\
 \Rightarrow k &\leq 4 & \frac{1}{2}
 \end{aligned}$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given quadratic equation is $x^2 + 4x + k = 0$.

Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = 1, b = 4, c = k$

Since, given the equation has real roots

$$\begin{aligned}
 \Rightarrow D &\geq 0 \\
 \Rightarrow b^2 - 4ac &\geq 0 \\
 \Rightarrow 4^2 - 4 \times 1 \times k &\geq 0 \\
 \Rightarrow 4k &\leq 16 \\
 \Rightarrow k &\leq 4
 \end{aligned}$$

15. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$. **[A] [CBSE OD Set-I, II, III, 2019]**

$$\text{Sol. } 2x^2 - 4x + 3 = 0$$

$$\begin{aligned}
 \Rightarrow D &= 16 - 24 = -8 \\
 \therefore \text{Equation has no real roots.} & & 1
 \end{aligned}$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

$$\begin{aligned}
 \text{Given: } 2x^2 - 4x + 3 &= 0 \\
 \text{On comparing above with } ax^2 + bx + c &= 0, \\
 \text{we get, } a = 2, b = -4, c = 3 \\
 \text{We shall find } D &= b^2 - 4ac \\
 \text{So, } D &= (-4)^2 - 4(2) \times (3) \\
 &= -8 < 0 \text{ or } (-ve)
 \end{aligned}$$

Hence, the given equation has no real roots.

COMMONLY MADE ERROR

- Students often make mistakes in analyzing the nature of roots as they get confused with the conditions.

ANSWERING TIP

- Understand the different conditions for nature of roots.

16. Find the values(s) of k for which the equation $x^2 + 5kx + 16 = 0$ has real and equal roots.

[U] [CBSE SQP, 2018]

$$\begin{aligned}
 \text{Sol. For roots to be real and equal, } b^2 - 4ac &= 0 & \frac{1}{2} \\
 \Rightarrow (5k)^2 - 4 \times 1 \times 16 &= 0 & \frac{1}{2} \\
 k &= \pm \frac{8}{5}
 \end{aligned}$$

[CBSE Marking Scheme, 2018]

17. Find the value(s) of k if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has real roots.

[U] [CBSE SQP, 2017]

Sol. If Discriminant of quadratic equation is equal to zero, or more than zero, then roots are real.

$$\text{Given, } 3x^2 - k\sqrt{3}x + 4 = 0$$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -k\sqrt{3} \text{ and } c = 4$$

Since, Discriminant, $D = b^2 - 4ac$
and for real roots $b^2 - 4ac \geq 0$

$$\Rightarrow (-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$$

$$\Rightarrow 3k^2 - 48 \geq 0$$

$$\Rightarrow k^2 - 16 \geq 0$$

$$\Rightarrow (k - 4)(k + 4) \geq 0$$

$$\therefore k \leq -4 \text{ and } k \geq 4.$$



Short Answer Type Questions-I

(2 Marks Each)

1. Solve for x : $\sqrt{2x+9} + x = 13$

[A] [CBSE OD Set-II, 2016]



Topper Answer, 2016

$$\begin{aligned} \sqrt{2x+9} + x &= 13 \\ \sqrt{2x+9} &= 13 - x \\ 2x+9 &= (13-x)^2 \quad \Rightarrow \\ 2x+9 &= 169 + x^2 - 26x \quad \Rightarrow \quad x^2 + 169 - 26x - 9 - 2x \\ &= 169 + x^2 - 2x - 26 - 9 \quad \Rightarrow \quad x^2 - 28x + 160 = 0 \\ x^2 - 2x - 26 + 169 - 9 &= 0 \quad \Rightarrow \quad x^2 - 20x - 8x + 160 = 0 \\ x^2 - 2x - 26 + 160 &= 0 \quad \Rightarrow \quad x(x-20) - 8(x-20) = 0 \\ x^2 - 2x + 134 &= 0 \quad \Rightarrow \quad (x-8)(x-20) = 0 \\ \text{either } x &= 8 \text{ or } x = 20 \end{aligned}$$

2. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b .
[C] + [A] [CBSE Delhi Set I, II, III, 2016]

Sol. Substituting $x = \frac{2}{3}$ in $ax^2 + 7x + b = 0$

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\Rightarrow 4a + 42 + 9b = 0$$

$$\Rightarrow 4a + 9b = -42 \quad \dots(i)$$

Again, substituting $x = -3$ in $ax^2 + 7x + b = 0$

$$9a - 21 + b = 0$$

$$\Rightarrow 9a + b = 21 \quad \dots(ii)$$

Solving (i) and (ii), we get

$$a = 3 \text{ and } b = -6$$

3. Solve the following quadratic equation for x :

$$4x^2 - 4a^2x + (a^4 - b^4) = 0$$

[U] [Delhi CBSE Term-II, 2015, Set-I, II, 2016]

Sol. Given, $4x^2 - 4a^2x + (a^4 - b^4) = 0$

Comparing with $Ax^2 + Bx + C = 0$, we get

Here, $A = 4, B = -4a^2$ and $C = (a^4 - b^4)$

$$\text{Since, } x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\text{Then, } x = \frac{-(-4a^2) \pm \sqrt{(-4a^2)^2 - 4 \times 4 \times (a^4 - b^4)}}{2 \times 4}$$

$$\Rightarrow x = \frac{4a^2 \pm \sqrt{16a^4 - 16a^4 + 16b^4}}{8}$$

$$\Rightarrow x = \frac{4a^2 \pm \sqrt{16b^4}}{8} \quad 1$$

$$\begin{aligned} \Rightarrow x &= \frac{4a^2 \pm 4b^2}{8} \\ &= \frac{a^2 \pm b^2}{2} \end{aligned}$$

$$\therefore x = \frac{a^2 + b^2}{2} \text{ or } x = \frac{a^2 - b^2}{2} \quad 1$$

[CBSE Marking Scheme, 2015]

4. A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

[A] [CBSE OD Set-I, 2016]

Sol. Let unit's digit and ten's digit of the two digit number be x and y respectively

\therefore Number is $10y + x$

According to question,

$$10y + x = 4(y + x)$$

$$\Rightarrow 10y + x = 4y + 4x$$

$$\Rightarrow 10y - 4y = 4x - x$$

$$\Rightarrow 6y = 3x$$

$$\Rightarrow 2y = x$$

...(i)

QUADRATIC EQUATIONS

Also, $10y + x = 3xy$... (ii)
 $\Rightarrow 10y + 2y = 3(2y)y$ [From eq (i)]
 $\Rightarrow 12y = 6y^2$
 $\Rightarrow 6y^2 - 12y = 0$
 $\Rightarrow 6y(y - 2) = 0$
 $\Rightarrow y = 0$ or $y = 2$
 Rejecting $y = 0$ as tens digit should not be zero for a two digit number.
 $\Rightarrow x = 4$
 \therefore Required number $= 10y + x$
 $\Rightarrow 10 \times 2 + 4 = 24$.

5. In a cricket match, Harbhajan took three wickets less than twice the number of wickets taken by Zahir.

[AI] 6. For what value of k , the given quadratic equation $kx^2 - 6x - 1 = 0$ has no real roots ?

The product of the number of wickets taken by these two is 20. Represent the above situation in the form of quadratic equation. [A] [CBSE Term-II, 2015]

Sol. Let the number of wickets taken by Zahir be x .

Then, the number of wickets taken by Harbhajan $= 2x - 3$ $\frac{1}{2}$

According to question, $x(2x - 3) = 20$ $\frac{1}{2}$

$$\Rightarrow 2x^2 - 3x = 20$$

\therefore Required quadratic equation is,

$$2x^2 - 3x - 20 = 0. \quad 1$$

[CBSE Marking Scheme, 2015]

[CBSE Delhi, 2019]



Topper Answer, 2019

Given, quadratic equation $\Rightarrow kx^2 - 6x - 1 = 0$.
 where $a = k$, $b = -6$, $c = -1$.

For no real roots (you imaginary roots), discriminant must be less than 0.

That is, $D < 0$

$$\Rightarrow b^2 - 4ac < 0$$

$$\Rightarrow (-6)^2 - 4 \times k \times (-1) < 0$$

$$\Rightarrow 36 + 4k < 0$$

$$\Rightarrow 4k < -36 \Rightarrow k < -9$$

$\therefore k$ should be less than -9. ($k = -10, -11, \dots$)

7. Find the value of k for which the roots of the quadratic equation $2x^2 + kx + 8 = 0$ will have the equal roots ?

[C] + [A] [CBSE Term-II OD Comptt., 2017]

Sol. For equal roots, $D = 0$

$$\therefore b^2 - 4ac = 0$$

$$\Rightarrow k^2 = 4 \times 2 \times 8$$

$$k^2 = 64$$

$$k = \pm\sqrt{64}$$

$$\therefore k = \pm 8$$

8. If 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q . [U] [CBSE SQP, 2016]

Sol. Since, 2 is the root of $x^2 + kx + 12 = 0$

$$\Rightarrow (2)^2 + 2k + 12 = 0$$

$$\Rightarrow 2k + 16 = 0$$

$$k = -8 \quad \frac{1}{2}$$

Putting, $k = -8$ in $x^2 + kx + q = 0$

$$\Rightarrow x^2 - 8x + q = 0 \quad \frac{1}{2}$$

For equal roots,

$$(-8)^2 - 4(1)q = 0 \quad \frac{1}{2}$$

$$\Rightarrow 64 - 4q = 0$$

$$\Rightarrow 4q = 64$$

$$\Rightarrow q = 16 \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2016]

9. Find k so that the quadratic equation $(k + 1)x^2 - 2(k + 1)x + 1 = 0$ has equal roots.

[R] [CBSE Term-II 2016]

[CBSE Term-II, Set- I, III, 2015]

Sol. Since, $(k + 1)x^2 - 2(k + 1)x + 1 = 0$ has equal roots.

$$D = 0$$

$$\text{i.e., } b^2 = 4ac \quad 1$$

$$\Rightarrow 4(k + 1)^2 = 4(k + 1)$$

$$\Rightarrow k^2 + 2k + 1 = k + 1$$

$$\Rightarrow k^2 + k = 0$$

$$\Rightarrow k(k + 1) = 0$$

$$\Rightarrow k = 0 \text{ or } -1 \quad 1$$

$k = -1$ does not satisfy the equation

So, $k = 0$.

[CBSE Marking Scheme, 2016]



Short Answer Type Questions-II

(3 Marks Each)

- AI** 1. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/h and time of flight increased by 30 minutes. Find the original duration of flight.

C + A [CBSE Delhi Set-I & OD Set-I, 2020]

Sol. Let original speed of flight be x km/h, then according to question,

$$\frac{600}{x-200} - \frac{600}{x} = 30 \text{ minutes}$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}} \right]$$

$$\Rightarrow 600 \left[\frac{1}{x-200} - \frac{1}{x} \right] = \frac{30}{60} \text{ hour}$$

$$\Rightarrow \frac{x-x+200}{x(x-200)} = \frac{1}{2 \times 600}$$

$$\Rightarrow \frac{200}{x^2 - 200x} = \frac{1}{1200}$$

$$\Rightarrow x^2 - 200x = 240000$$

$$\Rightarrow x^2 - 200x - 240000 = 0$$

Here, $a = 1$, $b = -200$ and $c = -240000$

$$\begin{aligned} \therefore x &= \frac{200 \pm \sqrt{40000 + 960000}}{2 \times 1} \\ &= \frac{200 \pm \sqrt{1000000}}{2} \\ &= \frac{200 \pm 1000}{2} \\ &= \frac{200+1000}{2}, \frac{200-1000}{2} \\ &= 600, -400 \end{aligned}$$

Since, speed cannot be negative, therefore

original speed = 600 km/h.

and original distance = 600 km

$$\begin{aligned} \therefore \text{Time} &= \frac{\text{original distance}}{\text{original speed}} \\ &= \frac{600 \text{ km}}{600 \text{ km/hr.}} \\ &= 1 \text{ h} \end{aligned}$$

Hence, the original duration of flight is 1 h.

- 2.** A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

A [CBSE Delhi Set-II, 2020]

Sol. Let the speed of the train = x km/h

Total distance covered by the train = 480 km

\therefore Time taken to cover the 480 km distance

$$= \frac{480}{x} \text{ h}$$

If the speed has decreased 8 km/h, i.e., $(x-8)$ km/h

Then, time taken to cover the 480 km distance

$$= \frac{480}{x-8} \text{ h}$$

According to question,

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$\Rightarrow 480 \left[\frac{x-x+8}{x(x-8)} \right] = 3$$

$$\Rightarrow \frac{8}{x^2-8x} = \frac{3}{480} = \frac{1}{160}$$

$$\Rightarrow x^2 - 8x - 1280 = 0$$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -8$ and $c = -1280$

$$\begin{aligned} \therefore x &= \frac{8 \pm \sqrt{64 + 4 \times 1280}}{2 \times 1} \\ &= \frac{8 \pm \sqrt{5184}}{2} \\ &= \frac{8 \pm 72}{2} \\ &= \frac{8+72}{2}, \frac{8-72}{2} \\ &= \frac{80}{2}, \frac{-64}{2} \\ &= 40, -32 \end{aligned}$$

Since, negative speed cannot be possible.

Hence, the original speed of the train = 40 km/h.

- AI** 3. Solve for x : $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$, $x \neq -4, -7$.

U [CBSE OD Set-I, 2020]

Sol. Given, $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$

$$\Rightarrow \frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$$

$$\Rightarrow \frac{3}{x^2+4x+7x+28} = \frac{11}{30}$$

$$\Rightarrow \frac{3}{x^2+11x+28} = \frac{11}{30}$$

$$\Rightarrow 11x^2 + 121x + 308 = 90$$

$$\Rightarrow 11x^2 + 121x + 218 = 0$$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 11$, $b = 121$ and $c = 218$

$$\begin{aligned}
 \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-121 \pm \sqrt{14641 - 9592}}{22} \\
 \Rightarrow x &= \frac{-121 \pm \sqrt{5049}}{22} \\
 &= \frac{-121 \pm 71.06}{22} \\
 \Rightarrow x &= \frac{-121 + 71.06}{22}, \frac{-121 - 71.06}{22} \\
 \Rightarrow x &= \frac{-49.94}{22}, \frac{-192.06}{22} \\
 \Rightarrow x &= -2.27, -8.73.
 \end{aligned}$$

4. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train.

[U] [CBSE OD Set-II, 2020]

Sol. Total distance of the journey = 600 km

Let speed of fast train = x km/h,

then speed of slow train = $(x - 10)$ km/h

According to question,

$$\frac{600}{x-10} - \frac{600}{x} = 3$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}} \right]$$

$$\Rightarrow 600 \left[\frac{x - x + 10}{(x-10)x} \right] = 3$$

$$\Rightarrow \frac{6000}{x^2 - 10x} = 3$$

$$\Rightarrow x^2 - 10x - 2000 = 0$$

$$\Rightarrow x^2 - 50x + 40x - 2000 = 0$$

$$\Rightarrow x(x - 50) + 40(x - 50) = 0$$

$$\Rightarrow (x - 50)(x + 40) = 0$$

Either, $x = 50$ or $x = -40$

\therefore speed can not be negative.

So, the speed of fast train = 50 km/h,

and the speed of slow train = $50 - 10 = 40$ km/h.

COMMONLY MADE ERROR

Some students do not know how to frame the equation. Some frame it correctly but fail to solve it.

ANSWERING TIP

Emphasis on solving quadratic equation based on application problems is necessary.

5. Solve for x : $x^2 + 5x - (a^2 + a - 6) = 0$

[A] [CBSE OD Set-II, 2019]

[CBSE Term-II Foreign Set-I, 2015]

Sol. Given,

$$x^2 + 5x - (a^2 + a - 6) = 0$$

Then,

$$x = \frac{-5 \pm \sqrt{25 + 4(a^2 + a - 6)}}{2} \quad 1$$

$$\left[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

$$= \frac{-5 \pm (2a + 1)}{2} \quad 1$$

$$= \frac{2a - 4}{2} \text{ or } \frac{-2a - 6}{2}$$

$$\text{Thus, } x = a - 2 \text{ or } x = -(a + 3) \quad \frac{1}{2} + \frac{1}{2}$$

[CBSE Marking Scheme, 2019]

Alternate Solution:

$$x^2 + 5x - (a^2 + a - 6) = 0$$

$$\Rightarrow x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$$

$$\Rightarrow x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0$$

$$\Rightarrow x^2 + 5x - (a + 3)(a - 2) = 0$$

$$\Rightarrow x^2 + [(a + 3) - (a - 2)]x - (a + 3)(a - 2) = 0$$

$$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$$

$$\Rightarrow x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$$

$$\Rightarrow [x + (a + 3)][x - (a - 2)] = 0$$

$$\Rightarrow x = -(a + 3) \text{ or } x = a - 2$$

Hence, roots of given equation are $-(a + 3)$ and $(a - 2)$.

6. Divide 27 into two parts such that the sum of their reciprocals is $\frac{3}{20}$.

[A] [CBSE Comptt. Set-I, II, III, 2018]

Sol. Let two parts be x and $27 - x$.

$$\therefore \frac{1}{x} + \frac{1}{27 - x} = \frac{3}{20}$$

$$\Rightarrow \frac{27 - x + x}{x(27 - x)} = \frac{3}{20}$$

$$\Rightarrow x^2 - 27x + 180 = 0$$

$$\Rightarrow (x - 15)(x - 12) = 0$$

$$\Rightarrow x = 12 \text{ or } 15.$$

\therefore The two parts are 12 and 15.

[CBSE Marking Scheme, 2018]

7. A plane left 30 minutes late then its scheduled time and in order to reach of destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

[A] [CBSE Delhi/OD Set-I, II, III, 2016, 2018]
[CBSE Term-II, 2015]

Sol. Let usual speed of the plane be x km/hr.

$$\therefore \frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60} \quad 1$$

$$\begin{aligned} \Rightarrow x^2 + 100x - 300000 &= 0 \\ \Rightarrow x^2 + 600x - 500x - 300000 &= 0 \quad 1 \\ \Rightarrow (x+600)(x-500) &= 0 \\ x = -600 \text{ or } x &= 500 \quad \frac{1}{2} \end{aligned}$$

(Rejecting negative value)

$$\text{Speed of plane} = 500 \text{ km/h} \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2018]

OR



Topper Answer, 2018

Given: distance is 1500 km.
Usual speed = s .
We know, speed = $\frac{\text{distance}}{\text{time}} \rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$.
 \rightarrow From question, $\frac{1500}{100+s} + \frac{1}{2} = \frac{1500}{s}$ [half an hour late].
(30 min = 0.5 hr).
 $\frac{1500}{100+s} = \frac{1500}{s} - \frac{1}{2}$
 $\frac{1500}{100+s} = \frac{3000-s}{2s}$
Cross multiplying,
 $3000s = 300000 - 100s + 500s - s^2$
 $s^2 + 100s - 300000 = 0$
 $s^2 + 600s - 500s - 300000 = 0$
 $s(s+600) - 500(s+600) = 0$
 $(s-500)(s+600) = 0$
 $\Rightarrow s-500=0 \text{ or } s+600=0$
 $\Rightarrow s=500 \text{ km/h or } s=-600 \text{ km/h}$
 $\Rightarrow s=500 \text{ or } -600 \text{ km/h}$
But speed cannot be negative.
 \Rightarrow The usual speed of the plane is 500 km/h.

[AI] 8. Solve for x :

$$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; \text{ where } x \neq 1, -2, 2$$

[U] [CBSE Term-II Delhi, Set-II, 2016]

$$\text{Sol. Here, } \frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2} \quad 1$$

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

$$\Rightarrow 5x^2 + 19x - 30 = 0 \quad 1$$

$$\Rightarrow (5x-6)(x+5) = 0$$

$$\Rightarrow x = -5 \text{ or } \frac{6}{5} \quad 1$$

[CBSE Marking Scheme, 2016]

Detailed Solution:

$$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$$

$$\frac{x+1}{x-1} - 1 + \frac{x-2}{x+2} - 1 = 2 - \frac{2x+3}{x-2}$$

$$\frac{x+1-x+1}{x-1} + \frac{x-2-x-2}{x+2} = \frac{2x-4-2x-3}{x-2}$$

$$\frac{2}{x-1} + \frac{-4}{x+2} = \frac{-7}{x-2}$$

$$\frac{2x+4-4x+4}{(x-1)(x+2)} = \frac{-7}{x-2}$$

$$\frac{-2x+8}{x^2+x-2} = \frac{-7}{x-2}$$

$$-2x^2 + 4x + 8x - 16 = -7x^2 - 7x + 14$$

$$\begin{aligned}
 5x^2 + 19x - 30 &= 0 \\
 5x^2 + 25x - 6x - 30 &= 0 \\
 5x(x + 5) - 6(x + 5) &= 0 \\
 (x + 5)(5x - 6) &= 0
 \end{aligned}$$

Either $x + 5 = 0 \Rightarrow x = -5$
 or $5x - 6 = 0 \Rightarrow x = \frac{6}{5}$

[AI] 9. Solve the following quadratic equation for x:

$$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a} \right) x + 1 = 0.$$

[U] [CBSE Term-II Delhi Set-III, 2016]

Sol. Here, $x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$

$$\Rightarrow x \left(x + \frac{a}{a+b} \right) + \frac{a+b}{a} \left(x + \frac{a}{a+b} \right) = 0 \quad 1$$

$$\Rightarrow \left(x + \frac{a}{a+b} \right) \left(x + \frac{a+b}{a} \right) = 0 \quad 1$$

$$\Rightarrow x = \frac{-a}{a+b} \text{ or } \frac{-(a+b)}{a} \quad 1$$

[CBSE Marking Scheme, 2016]

10. Solve the following quadratic equation for x:

$$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$$

[U] [CBSE Term-II Foreign Set-I, 2016]

Sol. Given,

$$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$$

First, we solve,

$$2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$$

Here, $= 2a[a + 2b] + b[a + 2b]$
 $= (a + 2b)(2a + b)$

Hence, the equation becomes

$$9x^2 - 9(a+b)x + (a+2b)(2a+b) = 0$$

$$\Rightarrow 9x^2 - 3[3a + 3b]x + (a+2b)(2a+b) = 0$$

$$\Rightarrow 9x^2 - 3[(a+2b) + (2a+b)]x + (a+2b)(2a+b) = 0$$

$$\Rightarrow 9x^2 - 3(a+2b)x - 3(2a+b)x + (a+2b)(2a+b) = 0$$

$$\Rightarrow 3x[3x - (a+2b)] - (2a+b)[3x - (a+2b)] = 0$$

$$\Rightarrow [3x - (a+2b)][3x - (2a+b)] = 0$$

$$\Rightarrow 3x - (a+2b) = 0 \text{ or } 3x - (2a+b) = 0$$

$$\Rightarrow x = \frac{a+2b}{3} \text{ or } x = \frac{2a+b}{3}$$

Hence, the roots = $\frac{a+2b}{3}, \frac{2a+b}{3}$

11. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers. **[A] [CBSE OD Set III, 2016]**

Sol. Let the three consecutive natural numbers be x,

x + 1 and x + 2.

$$\therefore (x+1)^2 = (x+2)^2 - (x)^2 + 60$$

$$\Rightarrow x^2 + 2x + 1 = x^2 + 4x + 4 - x^2 + 60$$

$$\Rightarrow x^2 - 2x - 63 = 0$$

$$\Rightarrow x^2 - 9x + 7x - 63 = 0$$

$$\Rightarrow x(x-9) + 7(x-9) = 0$$

$$\Rightarrow (x-9)(x+7) = 0$$

Thus, $x = 9 \text{ or } x = -7$

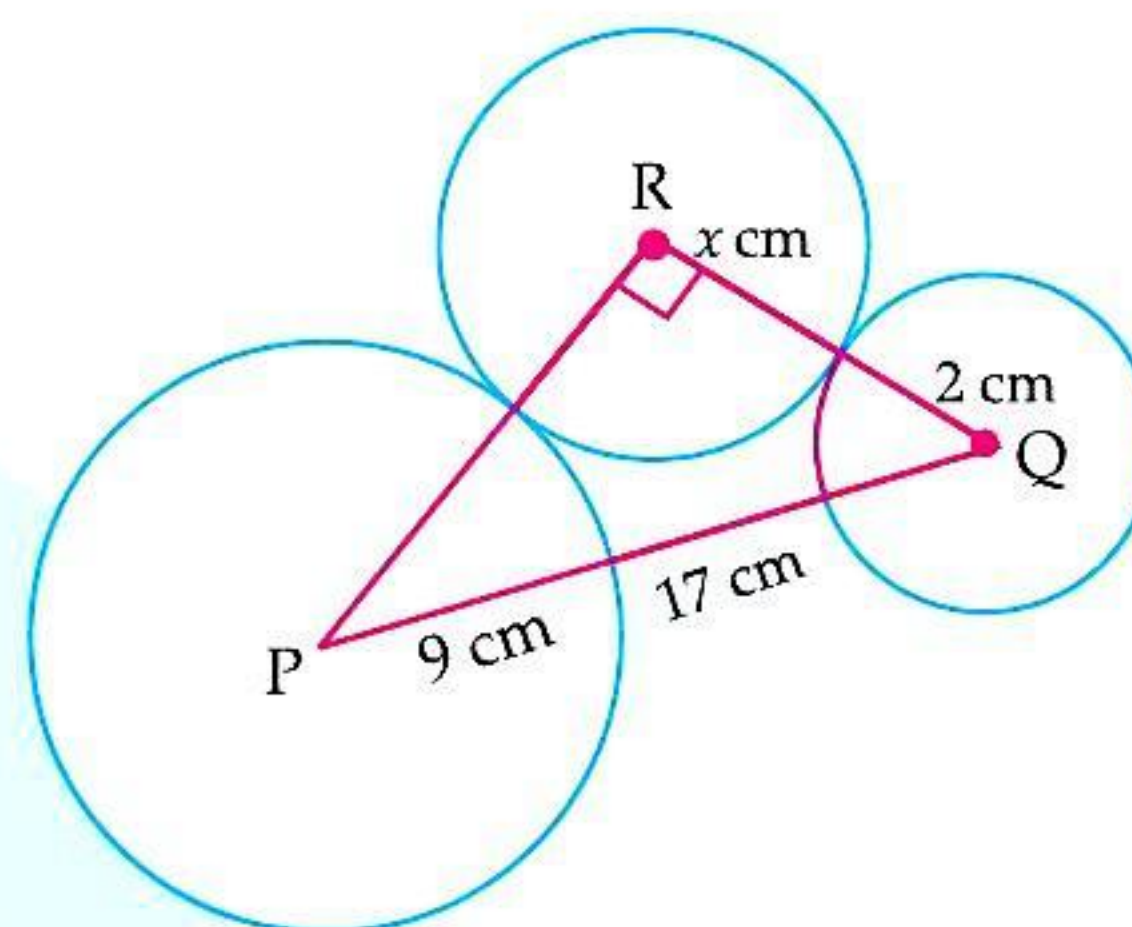
Rejecting -7, we get x = 9

Hence, three numbers are 9, 10 and 11.

12. P and Q are centres of circles of radii 9 cm and 2 cm respectively. PQ = 17 cm. R is the centre of the circle of radius x cm which touches given circles externally. Given that angle PRQ is 90°. Write an equation in x and solve it.

[A] [CBSE SQP, 2016]

Sol.



In right ΔPQR , by Pythagoras theorem

$$PQ^2 = PR^2 + RQ^2$$

$$\Rightarrow 17^2 = (x+9)^2 + (x+2)^2 \quad \frac{1}{2}$$

$$289 = x^2 + 18x + 81 + x^2 + 4x + 4$$

$$\Rightarrow 2x^2 + 22x - 204 = 0$$

$$\Rightarrow x^2 + 11x - 102 = 0 \quad \frac{1}{2}$$

$$\Rightarrow x^2 + 17x - 6x - 102 = 0$$

$$\Rightarrow x(x+17) - 6(x+17) = 0 \quad \frac{1}{2}$$

$$(x-6)(x+17) = 0$$

$$\Rightarrow x = 6 \text{ or } x = -17 \text{ (x can't be negative)} \quad \frac{1}{2}$$

Thus, $x = 6 \text{ cm}$

[CBSE Marking Scheme, 2016] $\frac{1}{2}$

13. Solve the quadratic equation $(x-1)^2 - 5(x-1) - 6 = 0$

[A] [CBSE Term-II, 2015]

Sol. Given, $(x-1)^2 - 5(x-1) - 6 = 0$

$$\Rightarrow x^2 - 2x + 1 - 5x + 5 - 6 = 0$$

$$\Rightarrow x^2 - 7x + 6 - 6 = 0$$

$$\Rightarrow x^2 - 7x = 0$$

$$\Rightarrow x(x-7) = 0$$

$$\therefore x = 0 \text{ or } 7.$$

[AI] 14. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained. **[A]**

[CBSE OD Set-I, II, III, 2019]

Sol. $x^2 + px + 16 = 0$ have equal roots if

$$D = p^2 - 4(16)(1) = 0 \quad 1$$

$$p^2 = 64 \Rightarrow p = \pm 8 \quad \frac{1}{2}$$

$$\therefore x^2 \pm 8x + 16 = 0 \Rightarrow (x \pm 4)^2 = 0 \quad 1$$

$$x \pm 4 = 0$$

$$\therefore \text{Roots are } x = -4 \text{ and } x = 4 \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given quadratic equation,

$$x^2 + px + 16 = 0 \quad \dots(i)$$

If this equation has equal roots, then discriminant value is zero.

$$\text{i.e., } D = b^2 - 4ac = 0 \quad \dots(ii)$$

Now, comparing the given quadratic equation with $ax^2 + bx + c = 0$,

we get $a = 1, b = p$ and $c = 16$

\therefore From eq (ii),

$$p^2 - 4 \times 1 \times 16 = 0$$

$$\Rightarrow p^2 = 64$$

$$\Rightarrow p = \pm 8$$

when $p = 8$,

$$\text{from eq (i), } x^2 + 8x + 16 = 0$$

$$\Rightarrow x^2 + 4x + 4x + 16 = 0$$

$$\Rightarrow x(x + 4) + 4(x + 4) = 0$$

$$\Rightarrow (x + 4)(x + 4) = 0$$

$$\Rightarrow x = -4, -4$$

Hence, roots are -4 and -4 .

when $p = -8$,

$$\text{from eq. (i), } x^2 - 8x + 16 = 0$$

$$\Rightarrow x^2 - 4x - 4x + 16 = 0$$

$$\Rightarrow x(x - 4) - 4(x - 4) = 0$$

$$\Rightarrow (x - 4)(x - 4) = 0$$

$$\Rightarrow x = 4, 4$$

Hence, the required roots are either $-4, -4$ or $4, 4$.

[AI] 15. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

[A] [CBSE OD Set-III, 2017]

[Delhi Comptt. Set-I, OD Set-III, 2017]



Topper Answer, 2017

$A = (a^2 + b^2), B = -2(ac + bd), C = (c^2 + d^2)$
as roots are equal,
 $D = B^2 - 4AC = 0$
 $B^2 = 4AC$
 $[-2(ac + bd)]^2 = 4(a^2 + b^2)(c^2 + d^2)$
 $(4a^2c^2 + 8abcd + 4b^2d^2) = 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$
 $2abcd = a^2d^2 + b^2c^2$
 $0 = a^2d^2 - 2abcd + b^2c^2$
 $0 = (ad - bc)^2$
 $0 = ad - bc$
 $ad = bc$
 $\Rightarrow \frac{a}{b} = \frac{c}{d}$
Hence, proved.

16. If $ad \neq bc$, then prove that the equation.

$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots. [CBSE OD Set-I 2017]

Sol. We have, $A = (a^2 + b^2), B = 2(ac + bd)$ and $C = (c^2 + d^2)$

For no real roots, $D < 0$

$$\text{i.e., } B^2 - 4AC < 0$$

$$\text{Now, } B^2 - 4AC = [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$$

$$= 4[a^2c^2 + 2abcd + b^2d^2]$$

$$- 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$

$$= 4[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$$

$$= -4[a^2d^2 + b^2c^2 - 2abcd]$$

$$= -4(ad - bc)^2$$

Since, $ad \neq bc$

Therefore, $D < 0$

Hence, the equation has no real roots.

17. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find k .

[C] + [A] [CBSE OD Comptt. Set-II, III, 2017]

Sol. Given, 2 is a root of the equation, $3x^2 + px - 8 = 0$

Putting $x = 2$ in $3x^2 + px - 8 = 0$, we get

$$12 + 2p - 8 = 0$$

$$\Rightarrow p = -2$$

Given, $4x^2 - 2px + k = 0$ has equal roots,

and $4x^2 + 4x + k = 0$ has equal roots.

$$\therefore b^2 - 4ac = 0$$

$$\Rightarrow (4)^2 - 4(4)(k) = 0$$

$$\Rightarrow 16 - 16k = 0$$

$$\Rightarrow 16k = 16$$

$$\therefore k = 1.$$

18. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

[CBSE OD Set-II, 2016]



Topper Answer, 2016

$$(a-b)x^2 + (b-c)x + (c-a) = 0$$

The roots are equal, then $D=0$

Comparing eqⁿ by $ax^2 + bx + c = 0$

$$a = (a-b); b = (b-c); c = (c-a)$$

$$D = b^2 - 4ac$$

$$= (b-c)^2 - 4(a-b)(c-a)$$

Here, $D = 0$

$$(b-c)^2 - 4(a-b)(c-a) = 0$$

$$b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$

$$b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$

$$4a^2 + b^2 + c^2 + 2bc - 4ab - 4ac = 0$$

$$\Rightarrow (-2a + b + c)^2 = 0 \quad [a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2]$$

$$-2a + b + c = 0$$

$$b + c = 2a$$

Hence proved

3



Long Answer Type Questions

(5 Marks Each)

[AI] 1. Solve the following equation:

$$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2 \quad \text{[CBSE SQP, 2020]}$$

Sol.

$$\frac{1}{x} - \frac{1}{x-2} = 3 \quad 1$$

$$\frac{x-2-x}{x(x-2)} = \frac{3}{1} \quad 1$$

$$3x^2 - 6x = -2$$

$$3x^2 - 6x + 2 = 0 \quad 1$$

$$x = \frac{6 \pm \sqrt{12}}{6} \quad 1$$

$$= \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3} \quad 1$$

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Given, $\frac{1}{x} - \frac{1}{x-2} = 3$

$$\Rightarrow \frac{x-2-x}{x(x-2)} = 3$$

$$\Rightarrow \frac{-2}{x^2 - 2x} = 3$$

$$\Rightarrow 3x^2 - 6x = -2$$

$$\Rightarrow 3x^2 - 6x + 2 = 0$$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 3, b = -6$ and $c = 2$

Now,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$= \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{3 \pm \sqrt{3}}{3}$$

Hence, $x = \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}$.

2. A train covers a distance of 360 km at a uniform speed. Had the speed been 5 km/h more, it would have taken 48 minutes less for the journey find the original speed of the train.

[A] [CBSE SQP, 2020]

Sol. Let the original speed of the train be x km/h

$$\therefore \frac{360}{x} - \frac{360}{x+5} = \frac{48}{60} \quad 2$$

$$\Rightarrow x^2 + 5x - 2250 = 0 \quad 1$$

$$\Rightarrow (x+50)(x-45) = 0 \therefore x = 45 \quad 1$$

$$\text{Hence original speed of the train} = 45 \text{ km/h} \quad 1$$

[CBSE SQP Marking Scheme, 2019]

Detailed Solution:

Let the original speed of the train be x km/h

$$\therefore \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken to cover a distance of 360 km,

$$t_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x} \text{ h}$$

Time taken to cover a distance of 360 km, when speed is increased by 5 km/h

$$t_2 = \frac{360}{x+5} \text{ h}$$

$$\text{Given, } t_1 - t_2 = 48 \text{ minutes}$$

$$= \frac{48}{60} = \frac{4}{5} \text{ h}$$

$$\text{Then, } \frac{360}{x} - \frac{360}{x+5} = \frac{4}{5}$$

$$\Rightarrow \frac{360x + 1800 - 360x}{x(x+5)} = \frac{4}{5}$$

$$\Rightarrow \frac{1800}{x^2 + 5x} = \frac{4}{5}$$

$$\Rightarrow \frac{450}{x^2 + 5x} = \frac{1}{5}$$

$$\Rightarrow x^2 + 5x - 2250 = 0$$

$$\Rightarrow x^2 + 50x - 45x - 2250 = 0$$

$$\Rightarrow x(x+50) - 45(x+50) = 0$$

$$\Rightarrow (x+50)(x-45) = 0$$

$$\text{If } x+50 = 0$$

$$\Rightarrow x = -50,$$

but speed can not be negative and if $x-45 = 0$

$$\Rightarrow x = 45$$

Hence, the speed of the train = 45 km/h.

[AI] 3. Two water taps together can fill a tank in $1\frac{7}{8}$

hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

[A] + [U] [CBSE Delhi Set-I, II, III, 2019, 2017]

[CBSE OD Set-III, Foreign Set-III, 2016]

Sol. Let the smaller tap fills the tank in x hrs

\therefore the larger tap fills the tank in $(x-2)$ hrs.

$$\text{Time taken by both the taps together} = \frac{15}{8} \text{ h}$$

$$\text{Therefore } \frac{1}{x} + \frac{1}{x-2} = \frac{8}{15} \quad 2$$

$$\Rightarrow 4x^2 - 23x + 15 = 0 \quad 1$$

$$\Rightarrow (4x-3)(x-5) = 0$$

$$x \neq \frac{3}{4} \therefore x = 5 \quad 1$$

Smaller and larger taps can fill the tank separately in 5 h and 3 h respectively. 1

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let the time taken by the smaller diameter tap

$$= x \text{ h.}$$

\therefore Time for larger diameter tap = $(x-2)$ h.

$$\text{Total time taken} = 1\frac{7}{8} = \frac{15}{8} \text{ h.}$$

Portion filled in one hour by smaller diameter tap

$$= \frac{1}{x}$$

$$\text{and by larger diameter tap} = \frac{1}{x-2}$$

According to the problem,

$$\Rightarrow \frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$$\Rightarrow \frac{x-2+x}{x(x-2)} = \frac{8}{15}$$

$$\Rightarrow 15(2x-2) = 8x(x-2)$$

$$\Rightarrow 30x-30 = 8x^2-16x$$

$$\Rightarrow 8x^2-46x+30 = 0$$

$$\Rightarrow 4x^2-23x+15 = 0$$

$$\Rightarrow 4x^2-20x-3x+30 = 0$$

$$\Rightarrow 4x(x-5)-3(x-5) = 0$$

$$\Rightarrow (4x-3)(x-5) = 0$$

$$\Rightarrow x = \frac{3}{4} \text{ or } x = 5$$

$$\text{If } x = \frac{3}{4}, \text{ then } x-2 = \frac{3}{4} - 2 = \frac{-5}{4}.$$

Since, time cannot be negative, we neglect $x = \frac{3}{4}$

Therefore, $x = 5$ and $x-2 = 5-2 = 3$

Hence, time taken by larger diameter tap = 3 hours

and time taken by smaller diameter tap = 5 hours.

4. In a class test, the sum of Arun's marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

[C] + [A] [CBSE OD Set-I, 2019]

Sol. Let marks in Hindi be x

Then marks in English = $30 - x$ 1

$$\therefore (x + 2)(30 - x - 3) = 210 \quad 1$$

$$\Rightarrow x^2 - 25x + 156 = 0 \text{ or } (x - 13)(x - 12) = 0 \quad 1$$

$$\Rightarrow x = 13 \text{ or } x = 12$$

$$\therefore 30 - 13 = 17 \text{ or } 30 - 12 = 18 \quad 1$$

\therefore Marks in Hindi & English are

(13, 17) or (12, 18) 1

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let the marks in Hindi be x

and the marks in English be y .

According to question,

$$x + y = 30$$

$$\Rightarrow y = 30 - x \quad \dots(i)$$

If he had got 2 marks more in Hindi, then his marks would be $= x + 2$

and if he had 3 marks less in English, then his marks would be $= y - 3$

According to question,

$$(x + 2)(y - 3) = 210$$

$$\Rightarrow (x + 2)(30 - x - 3) = 210 \quad [\text{from eq. (i)}]$$

$$\Rightarrow (x + 2)(27 - x) = 210$$

$$\Rightarrow 27x - x^2 + 54 - 2x = 210$$

$$\Rightarrow -x^2 + 25x - 156 = 0$$

$$\Rightarrow x^2 - 25x + 156 = 0$$

$$\Rightarrow x^2 - 13x - 12x + 156 = 0$$

$$\Rightarrow x(x - 13) - 12(x - 13) = 0$$

$$\Rightarrow (x - 12)(x - 13) = 0$$

$$\Rightarrow \text{Either } x = 12 \text{ or } x = 13$$

when $x = 12$,

$$\text{then } y = 30 - 12 = 18$$

when $x = 13$,

$$\text{then } y = 30 - 13 = 17$$

Hence, the marks in Hindi = 12 and marks in English = 18

or the marks in Hindi = 13 and marks in English = 17.

[AI] 5. The total cost of a certain length of a piece of cloth is ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre ?

[C] + [A] [CBSE OD Set-II, 2019]

Sol. Let total length of cloth = l m.

$$\therefore \text{Rate per meter} = ₹ \frac{200}{l} \quad 1$$

$$\Rightarrow (l + 5) \left(\frac{200}{l} - 2 \right) = 200 \quad 1$$

$$\Rightarrow (l + 5)(200 - 2l) = 200l$$

$$\Rightarrow l^2 + 5l - 500 = 0 \quad 1$$

$$\Rightarrow (l + 25)(l - 20) = 0$$

$$\Rightarrow l = 20 \quad 1$$

$$\therefore \text{Rate per metre} = ₹ \left(\frac{200}{20} \right) = ₹ 10 \text{ per meter} \quad 1$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let length of the cloth = x m

Cost of cloth per metre = ₹ y

Given, $x \times y = 200$

$$\Rightarrow y = \frac{200}{x} \quad \dots(i)$$

According to given conditions,

1. If the piece were 5 m longer

2. Each meter of cloth cost ₹ 2 less

$$\text{i.e., } (x + 5)(y - 2) = 200$$

$$\Rightarrow xy - 2x + 5y - 10 = 200$$

$$\Rightarrow xy - 2x + 5y = 210$$

$$\Rightarrow x \left(\frac{200}{x} \right) - 2x + 5 \left(\frac{200}{x} \right) = 210 \quad [\text{from eq. (i)}]$$

$$\Rightarrow 200 - 2x + \frac{1000}{x} = 210$$

$$\Rightarrow \frac{1000}{x} - 2x = 10$$

$$\Rightarrow 1000 - 2x^2 = 10x$$

$$\Rightarrow x^2 + 5x - 500 = 0$$

$$\Rightarrow x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow x(x + 25) - 20(x + 25) = 0$$

$$\Rightarrow (x + 25)(x - 20) = 0$$

$$\Rightarrow x = -25 \text{ or } x = 20$$

$$\therefore x = 20 \quad (\text{neglecting } x = -25)$$

$$y = \frac{200}{x} = \frac{200}{20} = 10$$

[From eq. (i)]

Hence, length of the piece of cloth is 20 m and original rate per meter is ₹ 10.

6. A shopkeeper buy certain number of books in ₹ 80. If he buy 4 more books then new cost price of each book is reduced by ₹ 1. Find the number of books initially he buy. [CBSE Delhi Term, 2019]



Topper Answer, 2019

28. Let the no. of books bought by the shopkeeper be 'n'.

Total money spent = ₹ 80

$$\therefore \text{Cost of each book} = \frac{\text{Rs } 80}{n}$$

Now, given: He buys 4 more books, no. of books bought = $n+4$
(for same amount)

$$\text{New cost of each book} = \frac{\text{Rs } 80}{n+4}$$

given, new cost of each book is Rs. 1 less than earlier.

$$\therefore \frac{80}{n} - \frac{80}{n+4} = 1$$

$$\Rightarrow 80 \left(\frac{1}{n} - \frac{1}{n+4} \right) = 1$$

$$\Rightarrow \frac{n+4-n}{n(n+4)} = \frac{1}{80} \Rightarrow 4 \times 80 = n(n+4)$$

$$\Rightarrow n^2 + 4n - 320 = 0$$

$$\text{Using quadratic formula; } \Rightarrow n = \frac{-4 \pm \sqrt{16 + 4 \times 320}}{2}$$

$$\Rightarrow n = \frac{-4 \pm \sqrt{16 + 1280}}{2}$$

$$= \frac{-4 \pm 26}{2} \Rightarrow n = \frac{-30}{2} \text{ or } \frac{22}{2}$$

$$\Rightarrow n = \frac{-4 + \sqrt{1296}}{2}$$

$$\Rightarrow n = \frac{-4 + 36}{2}$$

$$\Rightarrow n = \frac{-40}{2} \text{ or } \frac{32}{2} \Rightarrow n = -20 \text{ or } 16$$

Since, no. of books is a whole no., it cannot be negative.

$n = -20$ can be ignored.

$$\therefore \boxed{n = 16}$$

No. of books bought by the shopkeeper = 16.

7. A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete the total journey, what is the original average speed?

[C] + [A] [CBSE Delhi Set, 2018]

Sol. Let the original average speed of train be x km/hr.

$$\text{Therefore, } \frac{63}{x} + \frac{72}{x+6} = 3 \quad 1$$

$$x^2 - 39x - 126 = 0 \quad 1$$

$$(x-42)(x+3) = 0 \quad 1$$

x is not equal to -3 .

$$\therefore x = 42 \quad 1$$

Thus, original speed of train is 42 km/h. 1

[CBSE Marking Scheme, 2018]

Detailed Solution:

Let the original speed of train be x km/h

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Total time to complete journey = 3 h

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

$$\frac{63x + 378 + 72x}{x(x+6)} = 3$$

$$\frac{135x + 378}{x^2 + 6x} = 3$$

$$3x^2 + 18x = 135x + 378$$

$$3x^2 - 117x - 378 = 0$$

$$x^2 - 39x - 126 = 0$$

$$x^2 - 42x + 3x - 126 = 0$$

$$x(x-42) + 3(x-42) = 0$$

$$(x-42)(x+3) = 0$$

If $x+3=0 \Rightarrow x=-3$, speed cannot be negative.

If $x-42=0 \Rightarrow x=42 \therefore$ Speed of train = 42 km/h

8. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

[C] + [A] [CBSE Delhi/OD Set, 2018, 2017]
[CBSE Term-II, OD Set-II, 2016]

Sol. Let the speed of stream be x km/h.

Then, the speed of boat upstream = $(18-x)$ km/h

and speed of boat downstream = $(18+x)$ km/h 1

According to the question,

$$\Rightarrow \frac{24}{18-x} - \frac{24}{18+x} = 1 \quad 1$$

$$\Rightarrow \frac{24(18+x) - 24(18-x)}{18^2 - x^2} = 1$$

$$\Rightarrow 432 + 24x - 432 + 24x = 324 - x^2$$

$$\Rightarrow 48x = 324 - x^2$$

$$\Rightarrow x^2 + 48x - 324 = 0 \quad 1$$

$$\Rightarrow x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x+54) - 6(x+54) = 0$$

$$\Rightarrow (x+54)(x-6) = 0$$

$$\Rightarrow x+54=0 \quad \text{or} \quad x-6=0$$

$$\Rightarrow x=-54 \quad \text{or} \quad x=6 \quad 1$$

Since, speed cannot be negative.

Hence, the speed of steam $x = 6$ km/h. 1

[CBSE Marking Scheme, 2018]

Detailed Solution:



Topper Answer, 2018

23) Representative diagram:

(choice 1)

Given that:

Speed of boat = 18 km/h in still water.

Speed of stream = 5 (variable, must find)

Distance upstream = 24 km

Time upstream = 1 hr more than time downstream.

We know, $\text{Speed} \times \text{Time} = \text{Distance}$ $\Rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Time upstream = $\frac{24}{18-5}$, Time downstream = $\frac{24}{18+5}$.

$$\rightarrow \frac{24}{18-5} = 1 + \frac{24}{18+5}$$

$$\frac{24}{18-5} = \frac{18+5+24}{18+5}$$

(Cross-multiplying)

$$24(18+5) = (18+5)(18-5)$$

$$432+240 = 756+185-425-5^2$$

$$\rightarrow 5^2+245+240+432-756=0$$

$$5^2+485+(-324)=0$$

$$5^2+545-65-324=0$$

$$5(5+54)-6(5+54)=0$$

$$(5-6)(5+54)=0$$

Now, either $5-6=0$ or $5+54=0$

$$\rightarrow 5=6$$

$$\rightarrow 5=-54$$

So speed = 6 or -54 km/hr.

But speed cannot be negative.

\Rightarrow Speed of the stream is 6 km/hr.

AI 9. Solve $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, where $a+b \neq 0$.

A [CBSE SQP, 2018, 2016]

[CBSE Foreign Set-II, III 2017]

Sol. Given, $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

1

$$\Rightarrow \frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

1

Sol.

$$\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{a+b}{ab}$$

1

$$\Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

1

$$\Rightarrow x(a+b+x) = -ab$$

$$\Rightarrow x^2 + (a+b)x + ab = 0$$

$$\Rightarrow (x+a)(x+b) = 0$$

$$\Rightarrow x = -a \text{ or } x = -b$$

1

[CBSE Marking Scheme, 2018]

COMMONLY MADE ERROR

\Rightarrow Candidates do error in simplifying this type of equations.

ANSWERING TIP

\Rightarrow Adequate practice is necessary for simplifying this type of quadratic equations.

AI 10. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it?

A [CBSE OD Set-III, 2017]



Topper Answer, 2017

Let B complete a work in x days.

Then A takes $x-6$ days to complete it.

Together they complete it in 4 days.

According to work done per day,

$$\frac{1}{x-6} + \frac{1}{x} = \frac{1}{4}$$

$$\frac{x+x-6}{x(x-6)} = \frac{1}{4}$$

$$4(2x - 6) = x(x - 6)$$

$$8x - 24 = x^2 - 6x$$

$$\therefore x^2 - 14x + 24 = 0$$

$$x^2 - 12x - 2x + 24 = 0$$

$$x(x - 12) - 2(x - 12) = 0$$

$$(x - 2)(x - 12) = 0$$

$$\therefore x = 2 \text{ or } 12$$

$x = 2$ is not possible because then $x - 6$ is $\in \mathbb{N}$

$$\therefore x = 12$$

So, B takes 12 days to finish the work.

11. In a rectangular part of dimensions $50 \text{ m} \times 40 \text{ m}$ a rectangular pond is constructed so that the area of grass strip of uniform breadth surrounding the pond would be 1184 m^2 . Find the length and breadth of the pond.



[A] [CBSE Delhi Set-I, III 2017]

Sol. Let width of grass strip be $x \text{ m}$.

$$\therefore \text{Length of pond} = (50 - 2x) \text{ m}$$

$$\text{and breadth of pond} = (40 - 2x) \text{ m}$$

And area of park - area of pond = area of grass strip

$$\Rightarrow (50 \times 40) - (50 - 2x)(40 - 2x) = 1184$$

$$\Rightarrow 2000 - 2000 + 180x - 4x^2 = 1184$$

$$\Rightarrow x^2 - 45x + 296 = 0$$

$$\Rightarrow x^2 - 37x - 8x + 296 = 0$$

$$\Rightarrow x(x - 37) - 8(x - 37) = 0$$

$$\Rightarrow x = 8 \text{ or } 37$$

(37 is rejected, as it gives negatives values for length & breadth)

$$\text{Thus, the length of pond} = 50 - 2 \times 8 = 34 \text{ m}$$

$$\text{and breadth of pond} = 40 - 2 \times 8 = 24 \text{ m.}$$

12. Solve for x : $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

[U] [CBSE Delhi Set-I, II, III, 2016]

Sol. Given, $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y$

$$\therefore y^2 + 5y - 24 = 0$$

$$\Rightarrow (y + 8)(y - 3) = 0$$

$$y = 3 \text{ or } -8$$

Putting $y = 3$ we get

$$\frac{2x}{x-5} = 3$$

$$2x = 3x - 15$$

$$x = 15$$

\Rightarrow

Again, for $y = -8$, $\frac{2x}{x-5} = -8$

$$2x = -8x + 40$$

$$10x = 40$$

$$x = 4$$

$$x = 15 \text{ or } 4$$

Hence,

[CBSE Marking Scheme, 2016]

13. Find x in terms of a, b and c :

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c} \text{ where } x \neq a, b, c$$

[C] + [U] [CBSE Delhi Set-I, 2016]

Sol. $a(x-b)(x-c) + b(x-a)(x-c) = 2c(x-a)(x-b)$

$$\Rightarrow x^2(a+b-2c) + x(-ab-ac-ab-bc+2ac+2bc) = 0$$

$$\Rightarrow x^2(a+b-2c) + x(-2ab+ac+bc) = 0$$

$$\Rightarrow x = -\left(\frac{ac+bc-2ab}{a+b-2c}\right) \text{ and } x = 0$$

[CBSE Marking Scheme, 2016]

14. The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

[A] [CBSE Delhi Set-III, 2016]

Sol. Let numerator be x .

Then, the fraction = $\frac{x}{2x+1}$

Again, $\frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}$

$$\Rightarrow 21[x^2 + (2x+1)^2] = 58(2x^2 + x)$$

$$\Rightarrow 11x^2 - 26x - 21 = 0$$

$$11x^2 - 33x + 7x - 21 = 0$$

$$(x-3)(11x+7)=0$$

$$x=3 \text{ or } -\frac{7}{11} \quad 1$$

(rejected negative value)

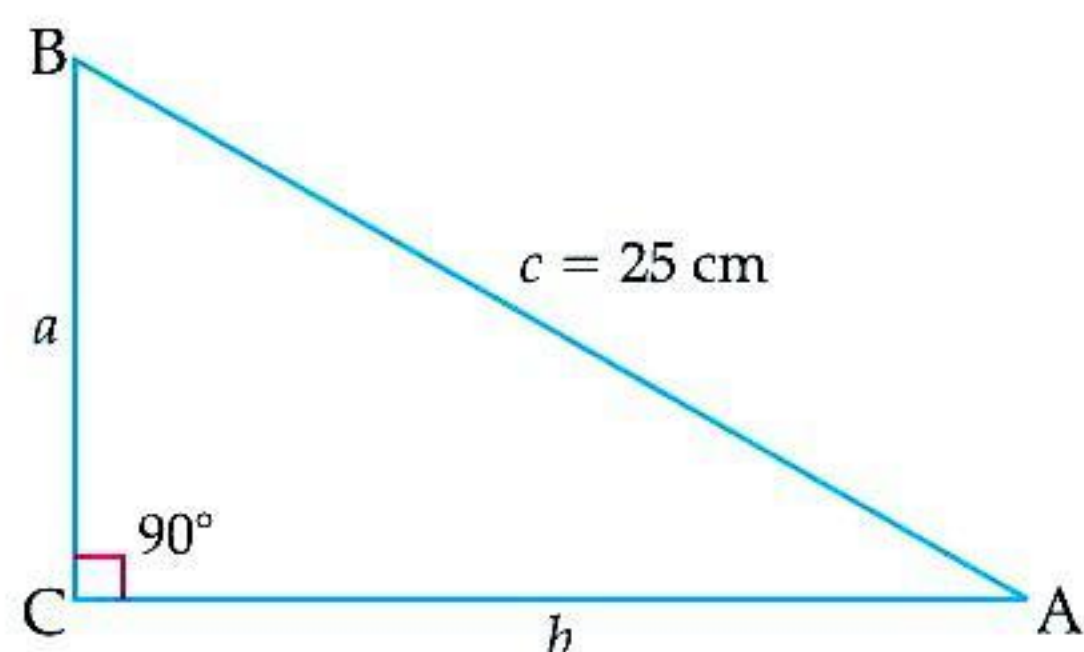
$$\text{Hence, fraction} = \frac{3}{7} \quad 1$$

[CBSE Marking Scheme, 2016]

15. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

[A] [CBSE Delhi Set-II, 2016]

Sol.



$$\text{Here, } a+b+c=60 \text{ and } c=25 \quad 1$$

$$a+b=60-c$$

$$a+b=60-25=35 \quad 1$$

Using Pythagoras theorem,

$$a^2+b^2=625$$

Using identity,

$$(a+b)^2=a^2+b^2+2ab \quad 1$$

$$35^2=625+2ab$$

$$\text{or, } 1225-625=2ab$$

$$\Rightarrow ab=300 \quad 1$$

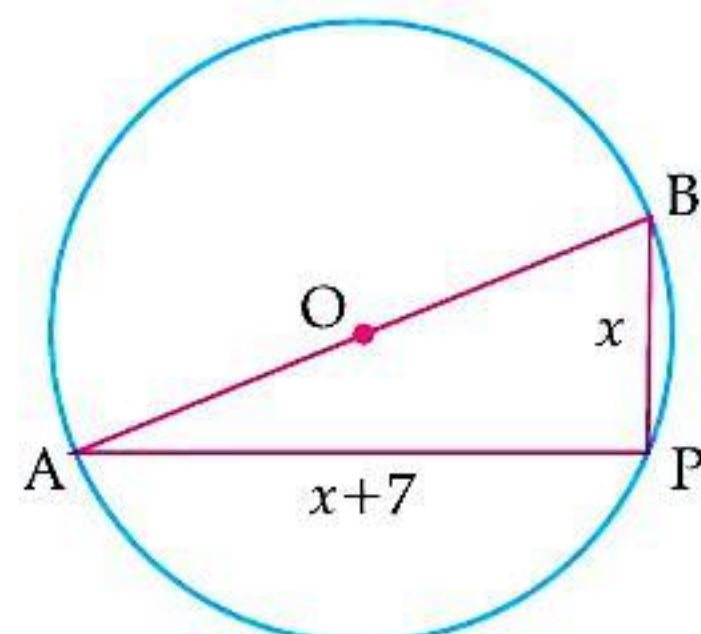
$$\text{Hence, area of } \triangle ABC = \frac{1}{2}ab = 150 \text{ cm}^2. \quad 1$$

[CBSE Marking Scheme, 2016]

16. A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Find the distances from the two gates where the pole is to be erected.

[CBSE Delhi Set-I, II, 2016]

Sol.



Let P be the location of the pole such that its distance from gate B, x metres. 1

$$\therefore AP = x+7 \quad 1$$

AB is diameter, $\angle APB = 90^\circ$ and $AB = 17 \text{ m}$ $\frac{1}{2}$

Using Pythagoras Theorem,

$$\therefore x^2 + (x+7)^2 = (17)^2$$

$$x^2 + x^2 + 14x - 240 = 0$$

$$\text{or } x^2 + 7x - 120 = 0 \quad 1\frac{1}{2}$$

$$x = \frac{-7 \pm \sqrt{49+480}}{2}$$

$$x = \frac{-7 \pm 23}{2} = 8, -15 \text{ (rejected)}$$

$$\therefore x = 8 \text{ m and } x+7 = 15 \text{ m} \quad 1$$

Hence, distance of the pole from the two gates = 8 m and 15 m. [CBSE Marking Scheme, 2016]

17. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return

journey. If he returned at a speed of 10 km/h more than the speed while going, find the speed in km/h in each direction. [A] [CBSE Delhi Set-III, 2016]

Sol. Let the speed while going be x km/h.

$$\therefore \text{Speed while returning} = (x+10) \text{ km/h}$$

According to question,

$$\frac{150}{x} - \frac{150}{x+10} = \frac{5}{2} \quad 2$$

$$\Rightarrow x^2 + 10x - 600 = 0$$

$$\Rightarrow (x+30)(x-20) = 0$$

$$\Rightarrow x = 20 \text{ or } -30 \quad 1$$

Rejecting negative value,

$$\therefore \text{Speed while going} = 20 \text{ km/h} \quad 1$$

$$\text{and speed while returning} = 20 + 10 = 30 \text{ km/h} \quad 1$$

[CBSE Marking Scheme, 2016]

[AI] 18. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the

original fraction. [A] [CBSE Delhi Set-I, III, 2015]

Sol. Let the denominator be x, then numerator = x-3

$$\text{So, the fraction} = \frac{x-3}{x} \quad 1$$

By the given condition,

$$\text{new fraction} = \frac{x-3+2}{x+2}$$

$$= \frac{x-1}{x+2} \quad 1$$

$$\text{Then, } \frac{x-3}{x} + \frac{x-1}{x+2} = \frac{29}{20}$$

$$\Rightarrow 20[(x-3)(x+2) + x(x-1)] = 29(x^2 + 2x)$$

$$\Rightarrow 20(x^2 - x - 6 + x^2 - x) = 29x^2 + 58x \quad 1$$

$$\Rightarrow 11x^2 - 98x - 120 = 0$$

$$\Rightarrow 11x^2 - 110x + 12x - 120 = 0$$

$$(11x+12)(x-10) = 0 \quad 1$$

$$\Rightarrow x = 10 \quad 1$$

$$\therefore \text{The fraction is } \frac{7}{10}. \quad \text{[CBSE Marking Scheme, 2015]} \quad 1$$

19. The diagonal of a rectangular field is 16 metre more than the shorter side. If the longer side is 14 metre more than the shorter side, then find the length of the sides of the field.

[C] + [A] [CBSE OD Set-I, II, III, 2015]

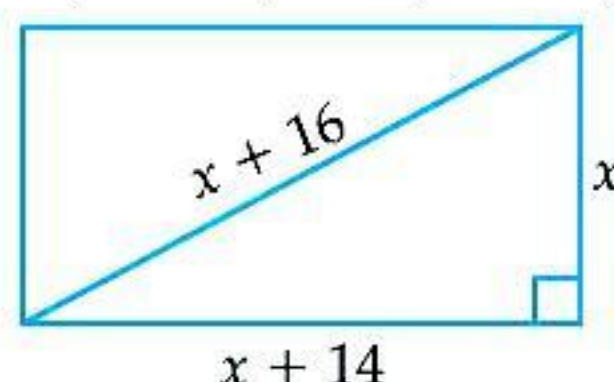
Sol. Let the length of shorter side be x m.

\therefore Length of diagonal = $(x + 16)$ m 1

and length of longer side = $(x + 14)$ m 1

Using Pythagoras Theorem,

$x^2 + (x + 14)^2 = (x + 16)^2$ 1



$\Rightarrow x^2 - 4x - 60 = 0$

$\Rightarrow x^2 + 6x - 10x - 60 = 0$

$\Rightarrow x(x + 6) - 10(x + 6) = 0$

$x = -6$ or $x = 10$

$\Rightarrow x = 10$ m 1

\therefore Length of sides are 10 m and 24 m. 1

[CBSE Marking Scheme, 2015]

20. Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots and if it has, find them by the method of completing the square. Also, verify that roots obtained satisfy the given equation.

[U] [CBSE SQP, 2017]

Sol. Discriminant $D = b^2 - 4ac$.

Here, $a = 5$, $b = (-6)$ and $c = (-2)$

Then, $b^2 - 4ac = (-6)^2 - 4 \times 5 \times -2$
 $= 36 + 40 = 76 > 0$

So the equation has real and two distinct roots.

Again, $5x^2 - 6x = 2$

$x^2 - \frac{6}{5}x = \frac{2}{5}$

(dividing both the sides by 5)

On adding square of the half of coefficient of x

$\Rightarrow x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{2}{5} + \frac{9}{25}$

$\Rightarrow x - \frac{3}{5} = \pm \frac{\sqrt{19}}{5}$

$\Rightarrow x = \frac{3 + \sqrt{19}}{5}$ or $\frac{3 - \sqrt{19}}{5}$

Verification:

$5 \left[\frac{3 + \sqrt{19}}{5} \right]^2 - 6 \left[\frac{3 + \sqrt{19}}{5} \right] - 2$

$= \frac{9 + 6\sqrt{19} + 19}{5} - \left(\frac{18 + 6\sqrt{19}}{5} \right) - 2$

$= \frac{28 + 6\sqrt{19}}{5} - \frac{18 + 6\sqrt{19}}{5} - 2$

$= \frac{28 + 6\sqrt{19} - 18 - 6\sqrt{19} - 10}{5}$

$= 0$

Similarly, we can verify that

$5 \left[\frac{3 - \sqrt{19}}{5} \right]^2 - 6 \left[\frac{3 - \sqrt{19}}{5} \right] - 2 = 0$

Hence Verified.

21. If the roots of the quadratic equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ in x are equal, then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

[U] [CBSE OD Set-II, III 2017]

Sol. Here, $A = (c^2 - ab)$, $B = -2(a^2 - bc)$, $C = (b^2 - ac)$

For real equal roots $D \Rightarrow B^2 - 4AC = 0$

$\Rightarrow [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow 4(a^4 + b^2c^2 - 2a^2bc) - 4(b^2c^2 - c^3a - ab^3 + a^2bc) = 0$

$\Rightarrow 4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + c^3a + ab^3 - a^2bc] = 0$

$\Rightarrow 4[a^4 + ac^3 + ab^3 - 3a^2bc] = 0$

$\Rightarrow a(a^3 + c^3 + b^3 - 3abc) = 0$

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$. Hence Proved.

22. The difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between the areas of the two circles is 1078 sq. cm. Find the radius of the smaller circle.

[U] [CBSE Comptt. I, II, III, 2017]

Sol. We have $r_2 - r_1 = 7$ cm, $r_2 > r_1$...(i) 1

and $\pi(r_2^2 - r_1^2) = 1078$ cm² ...(ii) 1

$\Rightarrow \pi(r_2 - r_1)(r_2 + r_1) = 1078$

$\Rightarrow r_2 + r_1 = \frac{1078}{22} = 49$...(iii) 1

On adding (i) and (iii) we get

$\Rightarrow 2r_2 = 56$

$\Rightarrow r_2 = 28$ cm

and $r_1 = 49 - 28 = 21$ 1

Hence, radii of smaller circle is 21 cm. 1

[CBSE Marking Scheme, 2017]

23. If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal.

[A] [CBSE Delhi Set-II, 2016]

Sol. For equal roots of $x^2 + 2px + mn = 0$, $4p^2 - 4mn = 0$

$\Rightarrow p^2 = mn$...(i) 2

For equal roots of

$x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$,

$4(m + n)^2 - 4(m^2 + n^2 + 2p^2) = 0$, 1

$m^2 + n^2 + 2mn - m^2 - n^2 - 2(mn) = 0$ [From (i)]

\therefore If roots of $x^2 + 2px + mn = 0$ are equal, then those of $x^2 - 2a(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal. [CBSE Marking Scheme, 2016] 2

24. Find the positive values of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have the real roots.

[C] + [A] [CBSE Delhi Set-I, 2016]

Sol. (i) For $x^2 + kx + 64 = 0$ to have real roots

$$k^2 - 256 \geq 0 \quad \dots(i) \quad 1\frac{1}{2}$$

(ii) For $x^2 - 8x + k = 0$ to have real roots

$$64 - 4k \geq 0 \quad \dots(ii) \quad 1\frac{1}{2}$$

For (i) and (ii) to hold simultaneously

$$k = 16 \quad 2$$

[CBSE Marking Scheme, 2016]

25. If (-5) is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k .

[CBSE Delhi Set-II, 2015]

Sol. Since, (-5) is a root of given quadratic equation

$$2x^2 + px - 15 = 0$$

$$\therefore 2(-5)^2 + p(-5) - 15 = 0 \quad 1$$

$$50 - 5p - 15 = 0$$

$$5p = 35 \Rightarrow p = 7 \quad 1$$

And $p(x^2 + x) + k = 0$ has equal roots

$$\Rightarrow px^2 + px + k = 0$$

$$\text{So, } (b)^2 - 4ac = 0$$

$$(p)^2 - 4p \times k = 0$$

$$(7)^2 - 4 \times 7 \times k = 0$$

$$28k = 49$$

$$k = \frac{49}{28} = \frac{7}{4} \quad 1$$

$$\text{Hence, } p = 7 \text{ and } k = \frac{7}{4}. \quad 1$$

[CBSE Marking Scheme, 2015]

26. If the roots of the quadratic equation

$$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$$

are equal. Then, show that $a = b = c$.

[CBSE Delhi Set-II, 2015]

Sol. Given,

$$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$$

$$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx$$

$$+ bc + x^2 - cx - ax + ac = 0$$

$$\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$$

$$\Rightarrow 3x^2 - 2(a+b+c)x + (ab+bc+ca) = 0$$

For equal roots, $B^2 - 4AC = 0$

$$\Rightarrow \{-2(a+b+c)\}^2 = 4 \times 3(ab+bc+ca)$$

$$\Rightarrow 4(a+b+c)^2 - 12(ab+bc+ca) = 0$$

$$\Rightarrow (a+b+c)^2 - 3(ab+bc+ca) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ac = 0$$

$$\Rightarrow a^2 + b^2 + c^2 - ab - ac - bc = 0$$

$$\Rightarrow \frac{1}{2} [2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc] = 0$$

$$\Rightarrow \frac{1}{2} [(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ac)] = 0$$

$$\Rightarrow \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$(a-b)^2 = 0, (b-c)^2 = 0 \text{ and } (c-a)^2 = 0$$

$$\text{if } (a-b)^2 = 0 \Rightarrow a = b$$

$$(a-c)^2 = 0 \Rightarrow b = c$$

$$(c-a)^2 = 0 \Rightarrow c = a$$

$$\therefore a = b = c \quad \text{Hence Proved.}$$