

Some Applications of Trigonometry

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 9.1

- Q. 1.** A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).

Sol. In the figure, let AC is the rope and AB is the pole. In right $\triangle ABC$, we have:

$$\frac{AB}{AC} = \sin 30^\circ$$

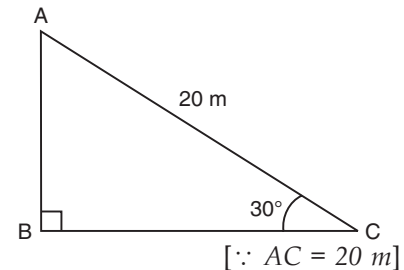
But $\sin 30^\circ = \frac{1}{2}$

$$\Rightarrow \frac{AB}{AC} = \frac{1}{2}$$

$$\Rightarrow \frac{AB}{20} = \frac{1}{2}$$

$$\Rightarrow AB = 20 \times \frac{1}{2} = 10 \text{ m}$$

Thus, the required height of the pole is **10 m**.



- Q. 2.** A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree. [CBSE 2012]

Sol. Let the original height of the tree = OP.

It is broken at A and its top is touching the ground at B.

Now, in right $\triangle AOB$, we have

$$\frac{AO}{OB} = \tan 30^\circ$$

But $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{AO}{OB} = \frac{1}{\sqrt{3}}$$

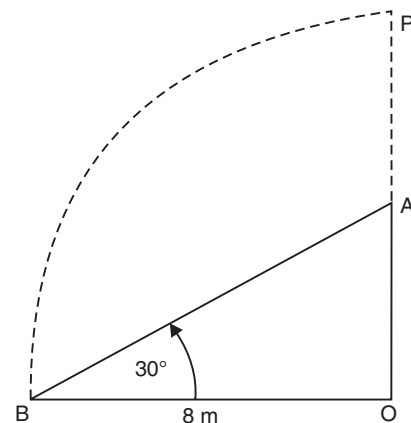
$$\Rightarrow \frac{AO}{8} = \frac{1}{\sqrt{3}} \Rightarrow AO = \frac{8}{\sqrt{3}}$$

Also, $\frac{AO}{OB} = \sec 30^\circ$

$$\Rightarrow \frac{AB}{8} = \frac{2}{\sqrt{3}} \Rightarrow AB = \frac{2 \times 8}{\sqrt{3}} = \frac{16}{\sqrt{3}}$$

Now, height of the tree

$$OP = OA + AP = OA + AB$$



$$\begin{aligned}
 &= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} & [\because AB = AP] \\
 &= \frac{24}{\sqrt{3}} \text{ m} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{ m} = 8\sqrt{3} \text{ m}
 \end{aligned}$$

Q. 3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for older children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Sol. In the figure, DE is the slide for younger children whereas AC is the slide for older children.

In right $\triangle ABC$,

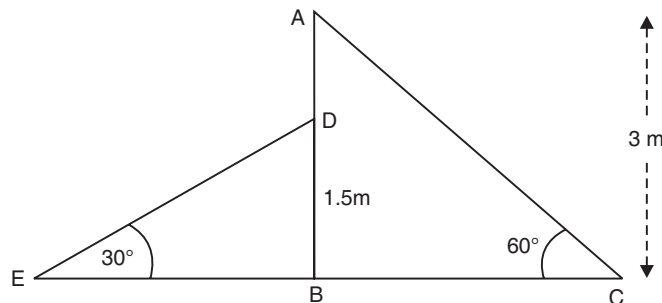
$$AB = 3 \text{ m}$$

AC = length of the slide

$$\therefore \frac{AB}{AC} = \sin 60^\circ$$

$$\Rightarrow \frac{3}{AC} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AC = \frac{2 \times 3}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$



Again in right $\triangle BDE$,

$$\frac{DE}{BD} = \operatorname{cosec} 30^\circ = 2$$

$$\Rightarrow \frac{DE}{1.5} = 2$$

$$\Rightarrow DE = 2 \times 1.5 \text{ m}$$

$$\Rightarrow DE = 3 \text{ m}$$

Thus, the lengths of slides are **3 m** and **$2\sqrt{3}$ m**.

Q. 4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30° . Find the height of the tower.

Sol. In right $\triangle ABC$, AB = the height of the tower. The point C is 30 m away from the foot of the tower,

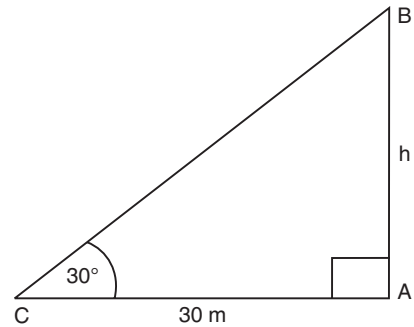
$$\therefore AC = 30 \text{ m}$$

$$\text{Now, } \frac{AB}{AC} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}} \quad [\because \tan 30^\circ = \frac{1}{\sqrt{3}}]$$

$$\Rightarrow h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$$

Thus, the required height of the tower is $10\sqrt{3} \text{ m}$.



- Q. 5.** A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string.

Sol. Let in the right ΔAOB ,

OB = Length of the string

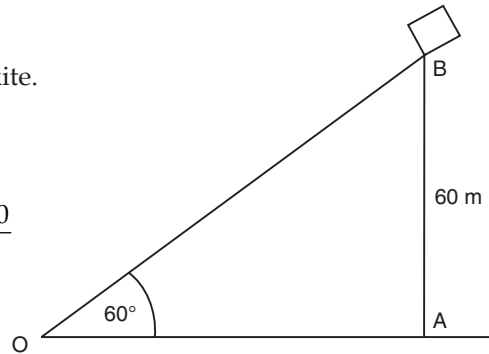
$AB = 60 \text{ m}$ = Height of the kite.

$$\therefore \frac{OB}{AB} = \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$$

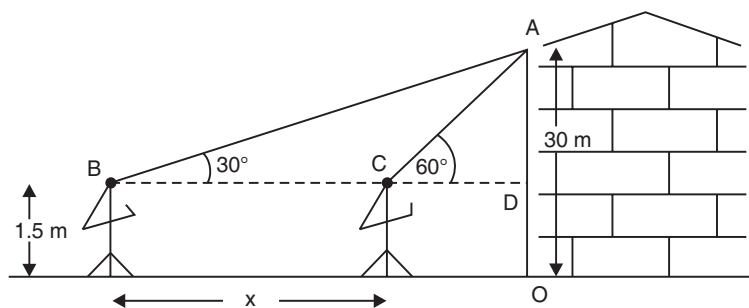
$$\Rightarrow \frac{OB}{60} = \frac{2}{\sqrt{3}} \Rightarrow OB = \frac{2 \times 60}{\sqrt{3}}$$

$$\Rightarrow OB = \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 40\sqrt{3}$$

Thus, length of the string is $40\sqrt{3} \text{ m}$.



- Q. 6.** A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.



Sol. Here, OA is the building.

In right ΔABD ,

$$\frac{AD}{BD} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BD = AD\sqrt{3} = 28.5\sqrt{3} \quad [\because AD = 30\text{ m} - 1.5\text{ m} = 28.5\text{ m}]$$

Also, in right ΔACD ,

$$\frac{AD}{CD} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow CD = \frac{AD}{\sqrt{3}} = \frac{28.5}{\sqrt{3}}$$

Now, $BC = BD - CD = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$

$$\Rightarrow BC = 28.5 \left[\sqrt{3} - \frac{1}{\sqrt{3}} \right]$$

$$= 28.5 \left[\frac{3-1}{\sqrt{3}} \right]$$

$$= 28.5 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{28.5 \times 2 \times \sqrt{3}}{3}$$

$$= 9.5 \times 2 \times \sqrt{3}$$

$$= 19\sqrt{3}\text{ m}$$

Thus the distance walked by the man towards the building = $19\sqrt{3}\text{ m}$.

- Q. 7.** From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. (CBSE 2010)

Sol. Let the height of the building be BC

$$\therefore BC = 20\text{ m}$$

And height of the tower be CD .

Let the point A be at a distance y metres from the top B of the building.

Now, in right ΔABC ,

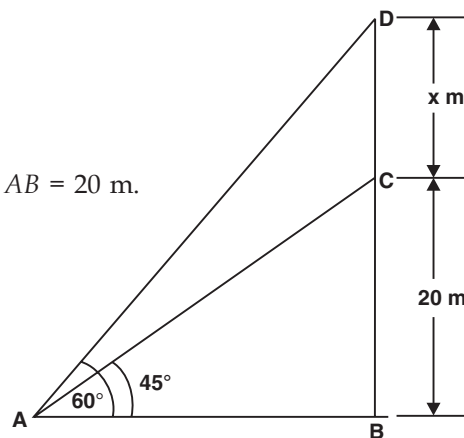
$$\frac{BC}{AB} = \tan 45^\circ = 1$$

$$\Rightarrow \frac{20}{y} = 1 \Rightarrow y = 20\text{ m} \text{ i.e., } AB = 20\text{ m.}$$

Now, in right ΔABD ,

$$\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \frac{BD}{20} = \sqrt{3}$$



$$\Rightarrow \frac{20+x}{20} = \sqrt{3} \Rightarrow 20+x = 20\sqrt{3}$$

$$\Rightarrow x = 20\sqrt{3} - 20 = 20[\sqrt{3} - 1]$$

$$\Rightarrow x = 20[1.732 - 1]$$

$$\Rightarrow x = 20 \times 0.732 = 14.64$$

Thus, the height of the tower is **14.64 m**.

- Q. 8.** A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal. (CBSE 2012)

Sol. In the figure,

DC represents the statue.

BC represents the pedestal.

Now in right $\triangle ABC$, we have

$$\frac{AB}{BC} = \cot 45^\circ = 1$$

$$\Rightarrow \frac{AB}{h} = 1 \Rightarrow AB = h \text{ metres.}$$

Now in right $\triangle ABD$, we get

$$\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow BD = \sqrt{3} \times AB = \sqrt{3} \times h$$

$$\Rightarrow h + 1.6 = \sqrt{3} h$$

$$\Rightarrow \frac{h+1.6}{h} = \sqrt{3} \Rightarrow h(\sqrt{3} - 1) = 1.6$$

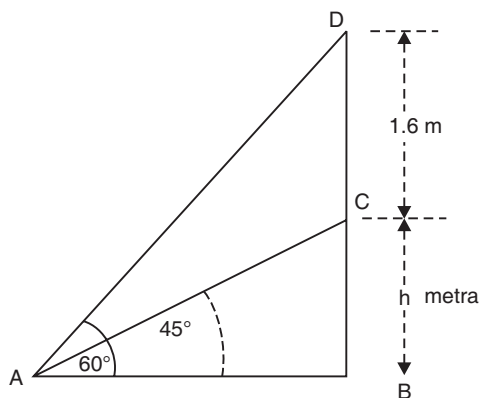
$$\Rightarrow h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$h = \frac{1.6}{3-1} \times (\sqrt{3}+1)$$

$$= \frac{1.6}{2} \times \sqrt{3} + 1$$

$$= 0.8(\sqrt{3} + 1) \text{ m}$$

Thus, the height of the pedestal = **$0.8(\sqrt{3} + 1)$ m**.



- Q. 9.** The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building. (CBSE Delhi 2014)

Sol. In the figure, let height of the building = $AB = h$ m

Let CD be the tower.

$$\therefore CD = 50 \text{ m}$$

Now, in right $\triangle BAC$,

$$\frac{AC}{AB} = \cot 30^\circ = \sqrt{3}$$

$$\Rightarrow \frac{AC}{h} = \sqrt{3} \Rightarrow AC = h\sqrt{3} \quad \dots(1)$$

Again, in right $\triangle DCA$,

$$\frac{DC}{AC} = \tan 60^\circ$$

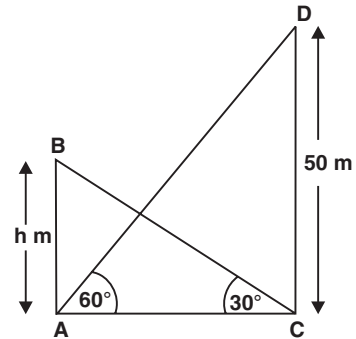
$$\Rightarrow \frac{50}{AC} = \sqrt{3} \Rightarrow AC = \frac{50}{\sqrt{3}} \quad \dots(2)$$

From (1) and (2),

$$\sqrt{3} h = \frac{50}{\sqrt{3}}$$

$$\Rightarrow h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3}$$

Thus, the height of the building = $16\frac{2}{3} \text{ m}$

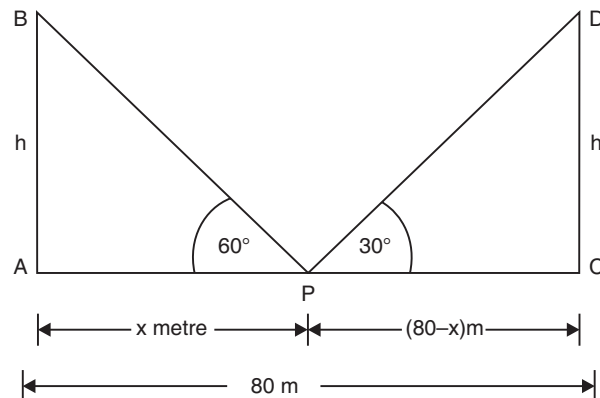


- Q. 10.** Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles. (CBSE 2012)

Sol. Let AB and CD are the two poles such that:

$$AB = h \text{ metres}$$

$$CD = h \text{ metres}$$



Let 'P' be the point on the road such that

$$AP = x \text{ m}$$

$$CP = (80 - x) \text{ m}$$

Now, in right ΔAPB , we have

$$\begin{aligned}\frac{AB}{AP} &= \tan 60^\circ \\ \Rightarrow \frac{h}{x} &= \sqrt{3} \Rightarrow h = x\sqrt{3} \quad \dots(1)\end{aligned}$$

Again in right ΔCPD ,

$$\begin{aligned}\frac{CD}{CP} &= \tan 30^\circ \\ \Rightarrow \frac{h}{(80-x)} &= \frac{1}{\sqrt{3}} \\ \Rightarrow h &= \frac{80-x}{\sqrt{3}} \quad \dots(2)\end{aligned}$$

From (1) and (2), we get

$$\begin{aligned}\sqrt{3}x &= \frac{80-x}{\sqrt{3}} \\ \Rightarrow \sqrt{3} \times \sqrt{3} \times x &= 80-x \\ \Rightarrow 3x &= 80-x \\ \Rightarrow 3x+x &= 80 \\ \Rightarrow 4x &= 80 \\ \Rightarrow x &= \frac{80}{4} = 20 \\ \Rightarrow 80-x &= 80-20 = 60\end{aligned}$$

Now, from (1), we have:

$$\begin{aligned}h &= \sqrt{3} \times 20 = 1.732 \times 20 \\ &= 34.64\end{aligned}$$

Thus, (i) The required point is **20 m** away from the first pole and **60 m** away from the second pole.

(ii) Height of each pole = **34.64 m**.

Q. 11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see figure). Find the height of the tower and the width of the canal.

Sol. Let the TV Tower be $AB = h$ m.

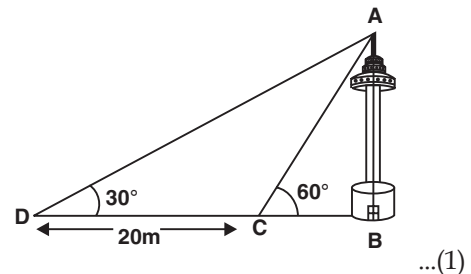
Let the point 'C' be such that

$$BC = x \text{ and } CD = 20 \text{ m.}$$

Now, in right ΔABC , we have:

$$\begin{aligned}\frac{AB}{BC} &= \tan 60^\circ \\ \Rightarrow \frac{h}{x} &= \sqrt{3} \Rightarrow h = \sqrt{3}x\end{aligned}$$

In right ΔABD , we have:



... (1)

$$\begin{aligned}\frac{AB}{BD} &= \tan 30^\circ \\ \Rightarrow \frac{h}{x+20} &= \frac{1}{\sqrt{3}} \\ \Rightarrow h &= \frac{x+20}{\sqrt{3}} \quad \dots(2)\end{aligned}$$

From (1) and (2), we get

$$\begin{aligned}\sqrt{3}x &= \frac{x+20}{\sqrt{3}} \Rightarrow 3x = x + 20 \\ \Rightarrow 3x - x &= 20 \\ \Rightarrow 2x &= 20 \Rightarrow x = \frac{20}{2} = 10 \text{ m}\end{aligned}$$

Now, from (1), we get

$$h = \sqrt{3} \times 10 = 1.732 \times 10 = 17.32$$

Thus, the height of the tower = **17.32 m.**

Also width of the river = **10 m.**

- Q. 12.** From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.

Sol. In the figure, let AB be the height of the tower.

$\therefore AB = 7$ metres.

Let CD be the cable tower.

\therefore In right $\triangle DAE$, we have

$$\begin{aligned}\frac{DE}{EA} &= \tan 60^\circ \\ \Rightarrow \frac{h}{x} &= \sqrt{3} \\ \Rightarrow h &= \sqrt{3} \cdot x \quad \dots(1)\end{aligned}$$

Again, in right $\triangle ABC$,

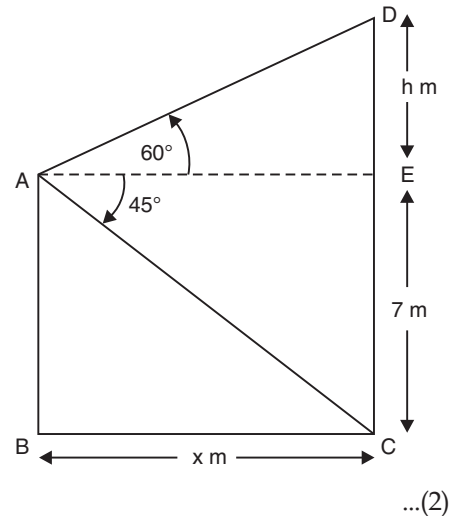
$$\begin{aligned}\frac{AB}{BC} &= \tan 45^\circ \\ \Rightarrow \frac{7}{x} &= 1 \\ \Rightarrow x &= 7\end{aligned} \quad \dots(2)$$

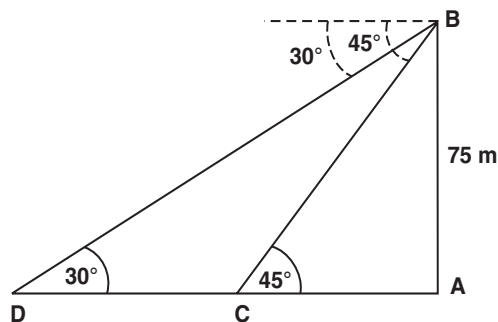
From (1) and (2),

$$\begin{aligned}h &= 7\sqrt{3} = DE \\ \therefore CD &= CE + ED \\ &= 7 + 7\sqrt{3} = 7(1 + \sqrt{3}) \text{ m} \\ &= 7(1 + 1.732) \text{ m} = 7 \times 2.732 \text{ m} = 19.124 \text{ m}\end{aligned}$$

Thus, the height of the cable tower is **19.124 m.**

- Q. 13.** As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.





Sol. In the figure, let AB represent the light house.

$\therefore AB = 75$ m.

Let the two ships be C and D such that angles of depression from B are 45° and 30° respectively.

Now in right $\triangle ABC$, we have:

$$\begin{aligned} \frac{AB}{AC} &= \tan 45^\circ \\ \Rightarrow \frac{75}{AC} &= 1 \Rightarrow AC = 75 \end{aligned} \quad \dots(1)$$

Again, in right $\triangle ABD$, we have:

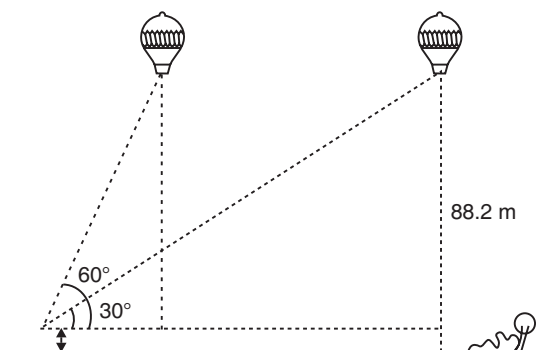
$$\begin{aligned} \frac{AB}{AD} &= \tan 30^\circ \\ \Rightarrow \frac{75}{AD} &= \frac{1}{\sqrt{3}} \Rightarrow AD = 75\sqrt{3} \end{aligned} \quad \dots(2)$$

Since the distance between the two ships = CD

$$\begin{aligned} &= AD - AC \\ &= 75\sqrt{3} - 75 = 75[\sqrt{3} - 1] \\ &= 75[1.732 - 1] = 75 \times 0.732 = 54.9 \end{aligned}$$

Thus, the required distance between the ships = **54.9 m.**

- Q. 14.** A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After some time, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval. (AI CBSE 2009)



Sol. In the figure, let C be the position of the observer (the girl).

A and P are two positions of the balloon.

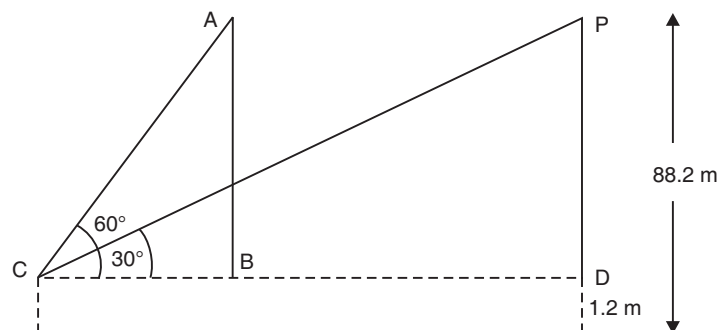
CD is the horizontal line from the eyes of the (observer) girl.

Here $PD = AB = 88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}$

In right $\triangle ABC$, we have

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}} \text{ m}$$



In right $\triangle PDC$, we have

$$\frac{PD}{CD} = \tan 30^\circ$$

$$\Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = 87\sqrt{3}$$

Now,

$$BD = CD - BC$$

$$= 87\sqrt{3} - \frac{87}{\sqrt{3}}$$

$$= 87 \left[\sqrt{3} - \frac{1}{\sqrt{3}} \right] = 87 \times \left(\frac{3-1}{\sqrt{3}} \right) = \frac{2 \times 87}{\sqrt{3}} \text{ m}$$

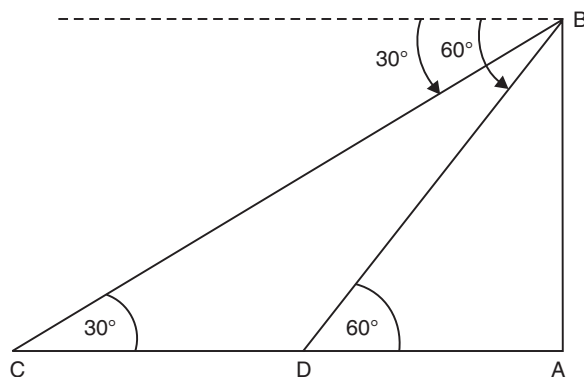
$$= \frac{2 \times 87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2 \times 87 \times \sqrt{3}}{3} = 2 \times 29 \times \sqrt{3} \text{ m}$$

$$= 58\sqrt{3} \text{ m}$$

Thus, the required distance between the two positions of the balloon = $58\sqrt{3} \text{ m}$

Q. 15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point. (CBSE 2009)

Sol. In the figure, let AB is the height of the tower and C and D be the two positions of the car.



In right ΔABD , we have:

$$\frac{AB}{AD} = \tan 60^\circ$$

$$\Rightarrow \frac{AB}{AD} = \sqrt{3} \Rightarrow AB = \sqrt{3} \cdot AD \quad \dots(1)$$

In right ΔABC , we have:

$$\frac{AB}{AC} = \tan 30^\circ$$

$$\Rightarrow \frac{AB}{AC} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{AC}{\sqrt{3}} \quad \dots(2)$$

From (1) and (2)

$$\sqrt{3} \cdot AD = \frac{AC}{\sqrt{3}}$$

$$\Rightarrow AC = \sqrt{3} \times \sqrt{3} \times AD = 3 AD$$

Now $CD = AC - AD$

$$= 3 AD - AD = 2 AD$$

Since the distance $2 AD$ is covered in 6 seconds,

\therefore The distance AD will be covered in $\frac{6}{2}$ i.e., 3 seconds.

Thus, the time taken by the car to reach the tower from D is **3 seconds**.

Q. 16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Sol. Let the tower be represented by AB in the figure.

Let $AB = h$ metres.

\therefore In right ΔABC , we have:

$$\frac{AB}{AC} = \tan \theta$$

$$\Rightarrow \frac{h}{9} = \tan \theta \quad \dots(1)$$

In right $\triangle ABD$, we have:

$$\frac{AB}{AD} = \tan (90^\circ - \theta) = \cot \theta$$

$$\Rightarrow \frac{h}{4} = \cot \theta \quad \dots(2)$$

Multiplying (1) and (2), we get

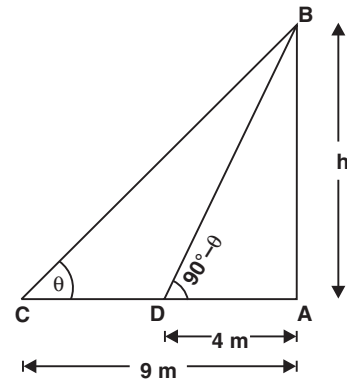
$$\frac{h}{9} \times \frac{h}{4} = \tan \theta \times \cot \theta = 1 \quad [\because \tan \theta \times \cot \theta = 1]$$

$$\Rightarrow \frac{h^2}{36} = 1 \Rightarrow h^2 = 36$$

$$\Rightarrow h = \pm 6 \text{ m}$$

$$\Rightarrow h = 6 \text{ m} \quad [\because \text{Height is positive only}]$$

Thus, the height of the tower is **6 m**.

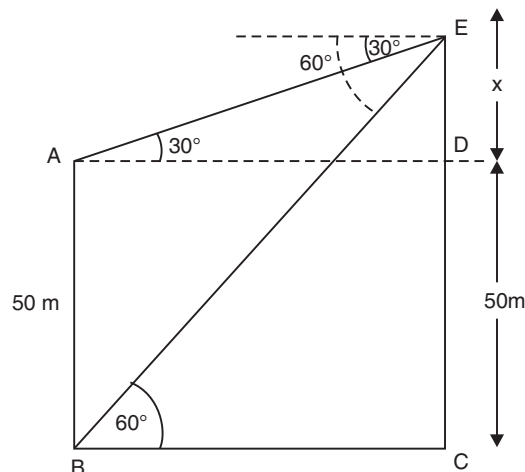


NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 9.2

- Q. 1.** The angles of depression of the top and the bottom of a building 50 m high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower and also the horizontal distance between the building and the tower.

Sol. In the figure,



Let $AB = 50$ m be the building.

Let CE be the tower such that $CE = (50 + x)$ m

In right $\triangle ADE$, we have:

$$\begin{aligned} \frac{DE}{AD} &= \tan 30^\circ = \frac{1}{\sqrt{3}} \\ \Rightarrow \frac{x}{AD} &= \frac{1}{\sqrt{3}} \Rightarrow AD = x\sqrt{3} \quad \text{or} \quad BC = x\sqrt{3} \end{aligned} \quad \dots(1)$$

In right $\triangle ACE$, we have:

$$\begin{aligned} \frac{CE}{BC} &= \tan 60^\circ = \sqrt{3} \\ \Rightarrow \frac{50+x}{BC} &= \sqrt{3} \Rightarrow BC = \frac{50+x}{\sqrt{3}} \end{aligned} \quad \dots(2)$$

From (1) and (2), we get

$$\begin{aligned} \sqrt{3}x &= \frac{50+x}{\sqrt{3}} \\ \Rightarrow \sqrt{3}x \times \sqrt{3} &= 50+x \\ \Rightarrow 3x - x &= 50 \Rightarrow x = 25 \end{aligned}$$

$$\begin{aligned} \therefore \text{Height of the tower} &= 50 + x \\ &= 50 + 25 \\ &= \mathbf{75 \text{ m}} \end{aligned}$$

$$\begin{aligned} \text{Now from (1), } BC &= \sqrt{3} \times x \\ &= \sqrt{3} \times 25 \text{ m} \\ &= 1.732 \times 25 \text{ m} \\ &= \mathbf{43.25 \text{ m}} \end{aligned}$$

i.e., The horizontal distance between the building and the tower = **43.25 m**.

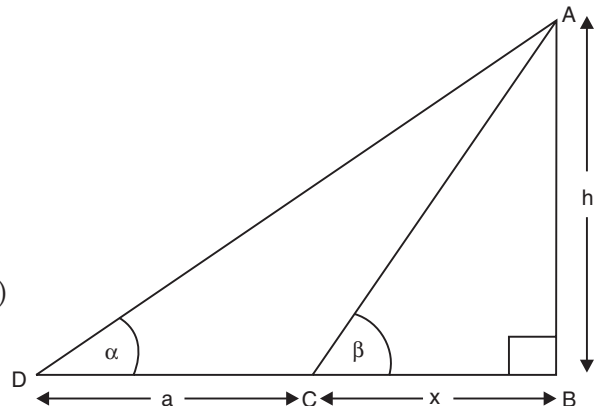
Q. 2. The angle of elevation of the top of a tower as observed from a point on the ground is ' α ' and on moving ' a ' metres towards the tower, the angle of elevation is ' β '. Prove that the height of the tower is $\frac{a \tan \alpha \cdot \tan \beta}{\tan \beta - \tan \alpha}$.

Sol. In the figure, let the tower be represented by AB .

\therefore In right $\triangle ABC$, we have:

$$\begin{aligned} \tan \beta &= \frac{AB}{BC} = \frac{h}{x} \\ \Rightarrow x \tan \beta &= h \\ \Rightarrow x &= \frac{h}{\tan \beta} \quad \dots(1) \end{aligned}$$

Now, in right $\triangle ABD$, we have:



$$\begin{aligned} \frac{AB}{BD} &= \tan \alpha \\ \Rightarrow \frac{h}{x+a} &= \tan \alpha \\ \Rightarrow h &= (x+a) \tan \alpha \\ \Rightarrow h &= x \tan \alpha + a \tan \alpha \\ \Rightarrow h &= \frac{h}{\tan \beta} \cdot \tan \alpha + a \tan \alpha & [\because x = \frac{h}{\tan \beta} \text{ from (1)}] \\ \Rightarrow h &= \frac{h \tan \alpha + a \tan \alpha \cdot \tan \beta}{\tan \beta} \\ \Rightarrow h \tan \beta &= h \tan \alpha + a \tan \alpha \cdot \tan \beta \\ \Rightarrow h \tan \beta - h \tan \alpha &= a \tan \alpha \cdot \tan \beta \\ \Rightarrow h (\tan \beta - \tan \alpha) &= a \tan \alpha \cdot \tan \beta \\ \Rightarrow h &= \frac{a \tan \alpha \cdot \tan \beta}{\tan \beta - \tan \alpha} \end{aligned}$$

Q. 3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 5 m. From a point on the plane the angles of elevation of the bottom and top of the flag staff are respectively 30° and 60° . Find the height of the tower.

Sol. Let in the figure, BC be the tower such that

$$BC = y \text{ metres.}$$

CD be the flag staff such that

$$CD = 5 \text{ m}$$

$$\Rightarrow BD = (y + 5) \text{ m.}$$

In right $\triangle ABC$, we have:

$$\frac{BC}{AB} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

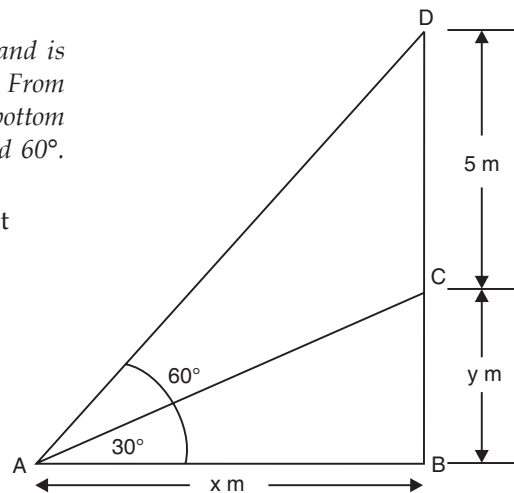
$$\Rightarrow \frac{y}{x} = \frac{1}{\sqrt{3}} \Rightarrow x = \sqrt{3} \cdot y \quad \dots(1)$$

In right $\triangle ABD$, we have:

$$\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \frac{(y+5)}{x} = \sqrt{3} \Rightarrow y + 5 = \sqrt{3} x$$

$$\therefore y + 5 = \sqrt{3} (\sqrt{3} \cdot y) \quad [x = \sqrt{3} \cdot y \text{ from (1)}]$$



$$\Rightarrow y + 5 = 3y$$

$$\Rightarrow 3y - y = 5 \Rightarrow y = \frac{5}{2} = 2.5 \text{ m}$$

\therefore The height of the tower = **2.5 m.**

Q. 4. The length of the shadow of a tower standing on level plane is found to be 20 m longer when the sun's altitude is 30° than when it was 60° . Find the height of the tower.

Sol. In the figure, let CD be the tower such that

$$CD = h \text{ metres}$$

$$\text{Also } BC = x \text{ metres}$$

In right $\triangle BCD$, we have:

$$\frac{CD}{BC} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow x = \frac{h}{\sqrt{3}} \quad \dots(1)$$

In right $\triangle ACD$, we have:

$$\frac{CD}{AC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{20+x} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3} h = 20 + x$$

$$\Rightarrow \sqrt{3} h = 20 + \frac{h}{\sqrt{3}} \quad [\text{From (1), } x = \frac{h}{\sqrt{3}}]$$

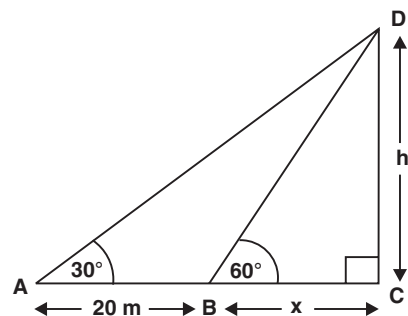
$$\Rightarrow \sqrt{3} \times \sqrt{3} h = 20\sqrt{3} + h$$

$$\Rightarrow 3h - h = 20\sqrt{3}$$

$$\Rightarrow 2h = 20\sqrt{3} \Rightarrow h = \frac{20}{2}\sqrt{3} = 10\sqrt{3}$$

$$\Rightarrow h = 10 \times 1.732 = 17.32 \text{ m}$$

Thus, the height of the tower = **17.32 m.**



Q. 5. From the top of a hill 200 m high, the angles of depression of the top and bottom of a pillar are 30° and 60° respectively. Find the height of the pillar and its distance from the hill. [CBSE 2014]

Sol. In the figure, let AD is the hill such that

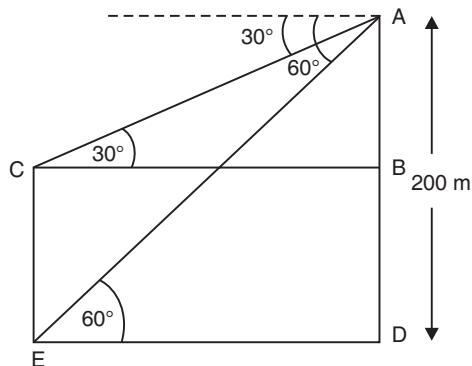
AD = 200 m and CE is the pillar.

In right $\triangle ADE$, we have:

$$\frac{AD}{DE} = \tan 60^\circ = \sqrt{3}$$

$$\therefore \frac{200}{DE} = \sqrt{3}$$

$$\Rightarrow DE = \frac{200}{\sqrt{3}} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$



$$\Rightarrow DE = \frac{\sqrt{3} \times 200}{3} = \frac{1.73 \times 200}{3}$$

$$= \frac{346}{3} = 115.33 \text{ m}$$

\Rightarrow Distance between pillar and hill = **115.33 m**

Now, $BC = DE = \frac{200}{\sqrt{3}} \text{ m}$ [$\because DE = BC$]

In right ΔABC , we have:

$$\frac{AB}{BC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{BC}{\sqrt{3}} = \frac{200}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{200}{3}$$

$$= 66.67 \text{ m}$$
[$\because BC = \frac{200}{\sqrt{3}}$]

\therefore Height of the pillar

$$CE = AD - AB$$

$$= 200 - 66.67 \text{ m}$$

$$= \mathbf{133.33 \text{ m}}$$
[$\because CE = BD$]

Q. 6. The angles of elevation of the top of a tower from two points on the ground at distances a and b units from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is \sqrt{ab} units.

Sol. In the figure, AB is the tower, such that:

$$AB = h$$

$$BD = b$$

$$BC = a$$

In right ΔABD , we have

$$\frac{AB}{BD} = \tan (90^\circ - \theta)$$

$$\Rightarrow \frac{h}{b} = \tan (90^\circ - \theta)$$

$$\Rightarrow h = b \cot \theta$$

In right ΔABC , we have

$$\frac{AB}{BC} = \tan \theta$$

$$\Rightarrow \frac{h}{a} = \tan \theta \Rightarrow h = a \tan \theta$$
...(2)

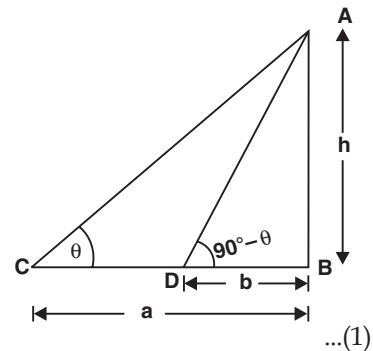
Multiplying (1) and (2), we get

$$h \times h = b \cot \theta \times a \tan \theta$$

$$\Rightarrow h^2 = a \times b \times (\cot \theta \times \tan \theta)$$

$$\Rightarrow h^2 = ab$$
[$\because \cot \theta \times \tan \theta = 1$]

$$\Rightarrow h = \sqrt{ab}$$



Q. 7. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression of the point 'A' from the top of the tower is 45° . Find the height of the tower. [A.I. CBSE 2004]

Sol. In the figure, let BC be the tower and CD be the pole.

Let $BC = x$ metres and $AB = y$ metres

In right $\triangle ABC$, we get

$$\frac{BC}{AB} = \tan 45^\circ = 1$$

$$\Rightarrow BC = AB \Rightarrow y = x \quad \dots (1)$$

In right $\triangle ABD$, we have:

$$\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \frac{x+5}{y} = \sqrt{3}$$

$$\Rightarrow y\sqrt{3} = x + 5$$

$$\Rightarrow x\sqrt{3} = x + 5$$

$$\therefore \sqrt{3}x - x = 5$$

$$\Rightarrow (\sqrt{3} - 1)x = 5$$

$$\Rightarrow x = \frac{5}{\sqrt{3} - 1} = \frac{5}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

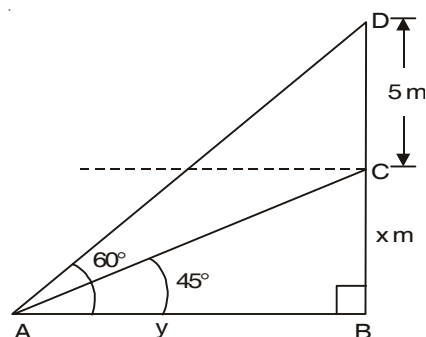
$$= \frac{5(\sqrt{3} + 1)}{3 - 1} = \frac{5(1.732 + 1)}{2}$$

$$= \frac{5}{2} \times 2.732 \text{ m}$$

$$= 5 \times 1.366 \text{ m}$$

$$= 6.83 \text{ m}$$

Thus, the height of the tower = 6.83 m



[$\because x = y$ from (1)]

MORE QUESTIONS SOLVED

I. SHORT ANSWER TYPE QUESTIONS

Q. 1. A tower stands vertically on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60° . Find the height of the tower. (CBSE 2010)

Sol. In the figure, AB is the tower,

$\therefore AB = h$ metres

In rt $\triangle ABC$, we have:

$$\frac{BC}{AC} = \tan 60^\circ$$

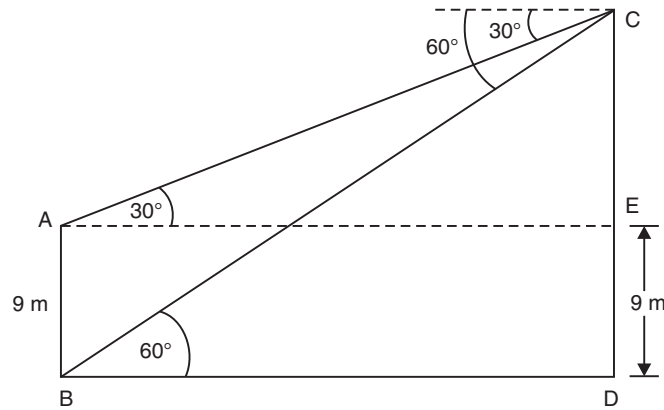
$$\Rightarrow \frac{h}{20} = \sqrt{3}$$

$$| \because \tan 60^\circ = \sqrt{3} \text{ and } AB = 20 \text{ m}$$

$$\Rightarrow h = 20\sqrt{3} \text{ metre}$$

Thus, the height of the tower = $20\sqrt{3}$ m.

Q. 2. The angle of depression of the top and the bottom of a 9 m high building from the top of a tower are 30° and 60° respectively. Find the height of the tower and the distance between the building and the tower.



Sol. Let AB represents the building and CD be the tower.

$$\therefore AB = 9 \text{ m}$$

In right $\triangle BDC$, we have:

$$\frac{CD}{DB} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow CD = DB \cdot \sqrt{3} \quad \dots(1)$$

In right $\triangle AEC$, we have:

$$\frac{CE}{AE} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{CD-9}{AE} = \frac{1}{\sqrt{3}} \Rightarrow AE = \sqrt{3} \quad CD - 9 \quad \sqrt{3}$$

$$\Rightarrow BD = \sqrt{3} \quad (DB \cdot \sqrt{3}) - 9\sqrt{3}$$

$$\Rightarrow BD = 3 \quad BD - 9\sqrt{3}$$

$$\Rightarrow 2BD = 9\sqrt{3}$$

$$\Rightarrow BD = \frac{9}{2}\sqrt{3} = \frac{9 \times 1.732}{2}$$

$$\Rightarrow BD = 7.8 \text{ m}$$

From (1), we have,

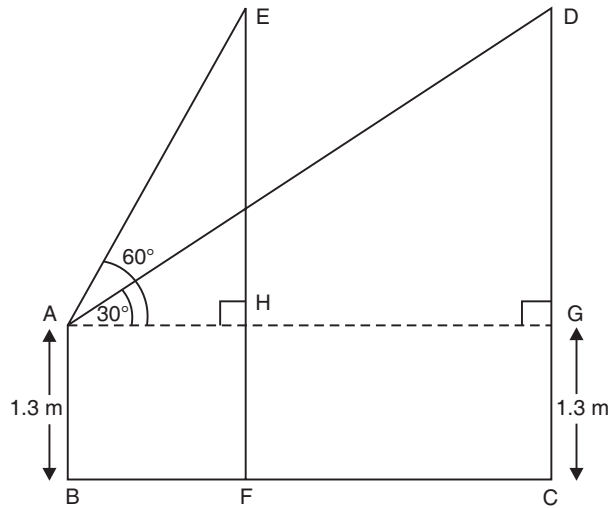
$$CD = \sqrt{3} \times \frac{9}{2} \times \sqrt{3} = \frac{27}{2} = 13.5$$

Thus, height of the tower = **13.5 m**

Distance between the building and the tower = **7.8 m**

II. LONG ANSWER TYPE QUESTIONS

- Q. 1.** A boy whose eye level is 1.3 m from the ground, spots a balloon moving with the wind in a horizontal level at some height from the ground. The angle of elevation of the balloon from the eyes of the boy at any instant is 60° . After 2 seconds, the angle of elevation reduces to 30° . If the speed of the wind at that moment is $29\sqrt{3}$ m/s, then find the height of the balloon from ground. (CBSE 2009 C)



Sol. Let E and D be the two positions of the balloon.

Let AB be the position of the boy.

$$\therefore AB = 1.3 \text{ m}$$

$$\Rightarrow HF = CG = 1.3 \text{ m}$$

Also speed of the wind = $29\sqrt{3}$ m/s

Distance covered by the balloon in 2 seconds

$$= ED = HG = 2 \times 29\sqrt{3} \text{ m}$$

$$= 58\sqrt{3} \text{ m}$$

$$\therefore AG = AH + HG$$

$$= AH + 58\sqrt{3} \text{ m}$$

...(1)

Now, in right $\triangle AEH$, we have

$$\frac{EH}{AH} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow EH = AH \cdot \sqrt{3} \Rightarrow AH = \frac{EH}{\sqrt{3}} \quad \dots(2)$$

In right $\triangle AGD$, we have

$$\frac{DG}{AG} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{DG}{(AH + 58\sqrt{3})} = \frac{1}{\sqrt{3}} \quad [\text{From (1)}]$$

$$\Rightarrow \sqrt{3} DG = AH + 58\sqrt{3}$$

$$\Rightarrow \sqrt{3} DG = \frac{EH}{\sqrt{3}} + 58\sqrt{3} \quad [\text{From (2)}]$$

$$\Rightarrow \sqrt{3} \times \sqrt{3} \times DG = EH + 58 \times \sqrt{3} \times \sqrt{3}$$

$$\Rightarrow 3 DG = EH + 3 \times 58$$

$$\Rightarrow 3 DG = EH + 174$$

$$\Rightarrow 3 DG - EH = 174$$

$$\Rightarrow 2 DG = 174 \quad [\because DG = EH]$$

$$\Rightarrow DG = \frac{174}{2} = 87 \text{ m}$$

$$\therefore CD = DG + GC = (87 + 1.3) \text{ m} \\ = 88.3 \text{ m}$$

Thus, the height of the balloon = **88.3 m**.

Q. 2. A statue, 1.5 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 45° and from the same point the angle of elevation of the top of the pedestal is 30° . Find the height of the pedestal from the ground. (CBSE 2012, 2009-C)

Sol. Let AB be the pedestal and $AB = h$

Let C be the point on the ground such that

$$BC = x \text{ metres.}$$

In right $\triangle ACB$, we have:

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$$

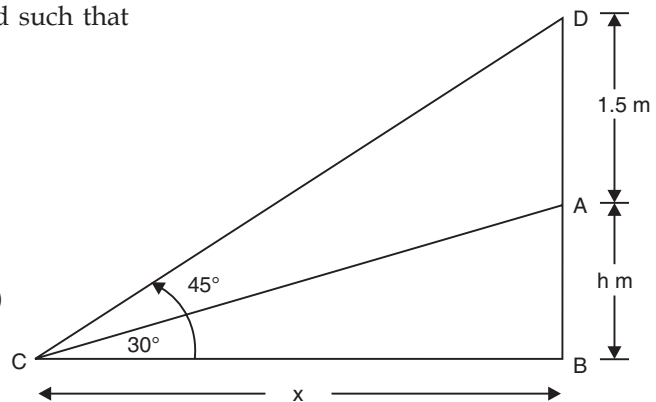
$$\Rightarrow x = \sqrt{3} h \dots(1)$$

In right $\triangle DCB$, we have:

$$\frac{BD}{BC} = \tan 45^\circ$$

$$\Rightarrow \frac{BD}{x} = 1$$

$$\Rightarrow \frac{AB + AD}{x} = 1$$



$$\begin{aligned}
\Rightarrow \quad & \frac{h+1.5}{x} = 1 \\
\Rightarrow \quad & h+1.5 = x \\
\Rightarrow \quad & h+1.5 = \sqrt{3}h \quad \text{[From (1)]} \\
\Rightarrow \quad & \sqrt{3}h - h = 1.5 \\
\Rightarrow \quad & h(\sqrt{3} - 1) = 1.5 \\
\Rightarrow \quad & h = \frac{1.5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\
\Rightarrow \quad & h = \frac{1.5(\sqrt{3}+1)}{3-1} = \frac{1.5(\sqrt{3}+1)}{2} \text{ m} \\
\Rightarrow \quad & h = 0.75(\sqrt{3}+1) \text{ m}
\end{aligned}$$

Thus, the height of the pedestal = $0.75(\sqrt{3}+1)$ m.

- Q. 3.** The angles of depression of the top and bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° , respectively. Find the height of the multi-storeyed building and the distance between the two buildings. (CBSE 2009)

Sol. Let the multistoreyed building be AB.

$$\therefore AB = q \text{ metres}$$

$$\Rightarrow AD = (q-8) \text{ m } [\because BD = 8 \text{ m}]$$

Let EC be the small building.

Now, in right $\triangle ABC$, we have:

$$\frac{AB}{BC} = \tan 45^\circ = 1$$

$$\Rightarrow AB = BC$$

$$\Rightarrow q = p \quad \dots(1)$$

In right $\triangle ADE$, we have:

$$\frac{AD}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3} AD = DE$$

$$\Rightarrow \sqrt{3}(q-8) = p$$

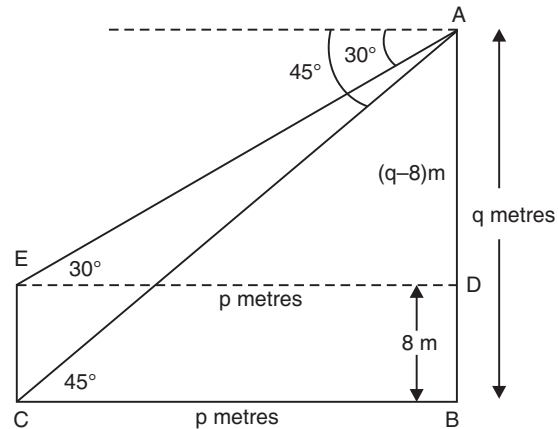
$$\Rightarrow \sqrt{3}q - 8\sqrt{3} = p \quad \text{[From (1)]}$$

$$\Rightarrow \sqrt{3}q - q = 8\sqrt{3}$$

$$\Rightarrow q(\sqrt{3}-1) = 8\sqrt{3}$$

$$\therefore q = \frac{8\sqrt{3}}{\sqrt{3}-1}$$

$$\Rightarrow q = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \text{ m}$$



$$\begin{aligned}
 &= \frac{8\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2} \text{ m} = \frac{8\sqrt{3}(\sqrt{3}+1)}{2} \text{ m} \\
 &= 4(3 + \sqrt{3}) \text{ m} = 4(3 + 1.732) \text{ m} \\
 &= 18.928 \text{ m}
 \end{aligned}$$

Since $p = q$
 $\Rightarrow p = 18.928 \text{ m}$

\therefore Distance between the two buildings = **18.928 m**

Height of the multi-storeyed building = **18.928 m**.

Q. 4. From the top of a building 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60° respectively. Find:

(i) The horizontal distance between the building and the lamp post.

(ii) The height of the lamp post. [Take $\sqrt{3} = 1.732$] (CBSE 2012)

Sol. In the figure, let CE be the building and AB be the lamp post

$\therefore CE = 60 \text{ m}$

In right $\triangle BCE$, we have:

$$\begin{aligned}
 \Rightarrow \frac{CE}{BC} &= \tan 60^\circ = \sqrt{3} \\
 \Rightarrow \frac{60}{BC} &= \sqrt{3} \\
 \Rightarrow BC &= \frac{60}{\sqrt{3}} = \frac{60 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \text{ m} \\
 \Rightarrow BC &= \frac{60\sqrt{3}}{3} = 20\sqrt{3} \text{ m}
 \end{aligned}$$

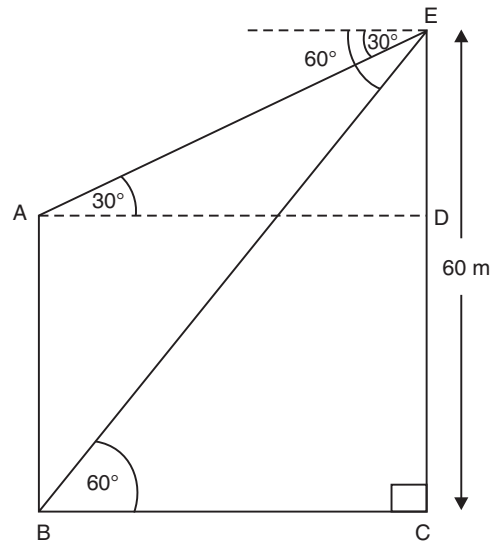
In right $\triangle ADE$, we have:

$$\begin{aligned}
 \Rightarrow \frac{DE}{AD} &= \tan 30^\circ = \frac{1}{\sqrt{3}} \\
 \Rightarrow \frac{DE}{20\sqrt{3}} &= \frac{1}{\sqrt{3}} \\
 [\because BC = AD = 20\sqrt{3} \text{ m}] \\
 \Rightarrow DE &= \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Height of the lamp post} &= AB = CD \\
 &= CE - DE \\
 &= 60 \text{ m} - 20 \text{ m} \\
 &= \mathbf{40 \text{ m.}}
 \end{aligned}$$

Also, the distances between the lamp post and the building

$$\begin{aligned}
 &= 20\sqrt{3} \text{ m} \\
 &= 20 \times 1.732 \text{ m} \\
 &= \mathbf{34.64 \text{ m}}
 \end{aligned}$$



$$[\because \sqrt{3} = 1.732]$$

- Q. 5.** The angle of elevation of a cloud from a point h meters above the surface of a lake is θ and the angle of depression of its reflection in the lake is ϕ . Prove that the height of the clouds above the lake is $h \left[\frac{\tan \phi + \tan \theta}{\tan \phi - \tan \theta} \right]$. [NCERT Exemplar]

Sol. Let P be the cloud and Q be its reflection in the lake. As shown in the figure, let A be the point of observation such that $AB = h$

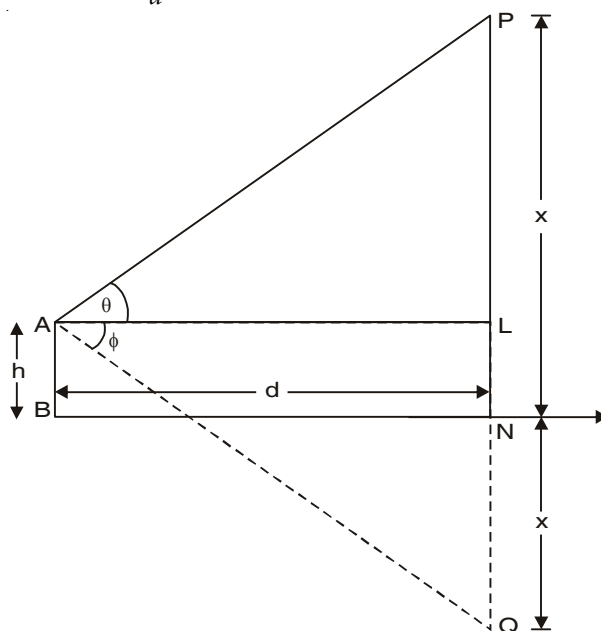
Let the height of the cloud above the lake = x

Let $AL = d$

$$\text{From rt } \triangle PLA, \tan \theta = \frac{PL}{AL} = \frac{PN - LN}{AL}$$

$$\Rightarrow \tan \theta = \frac{x - h}{d} \quad \dots(1)$$

$$\text{similarly, } \tan \phi = \frac{x + h}{d} \quad \dots(2)$$



$$\text{From (1) and (2), } \frac{\tan \phi}{\tan \theta} = \frac{x + h}{x - h}$$

$$\text{or } \frac{2x}{2h} = \frac{\tan \phi + \tan \theta}{\tan \phi - \tan \theta} \Rightarrow x = h \left[\frac{\tan \phi + \tan \theta}{\tan \phi - \tan \theta} \right]$$

- Q. 6.** From a point 100 m above a lake, the angle of elevation of a stationary helicopter is 30° and the angle of depression of reflection of the helicopter in the lake is 60° . Find the height of the helicopter. (AI CBSE 2008 C)

Sol. In the figure, A is the stationary helicopter and F is its reflection in the lake.

In right $\triangle AED$, we have:

$$\tan 30^\circ = \frac{AE}{DE}$$

$$\begin{aligned} \text{But } \tan 30^\circ &= \frac{1}{\sqrt{3}} \\ \therefore \frac{AE}{DE} &= \frac{1}{\sqrt{3}} \\ \Rightarrow \frac{x-100}{y} &= \frac{1}{\sqrt{3}} \\ \Rightarrow y &= \frac{x-100}{\sqrt{3}} \quad \dots(1) \end{aligned}$$

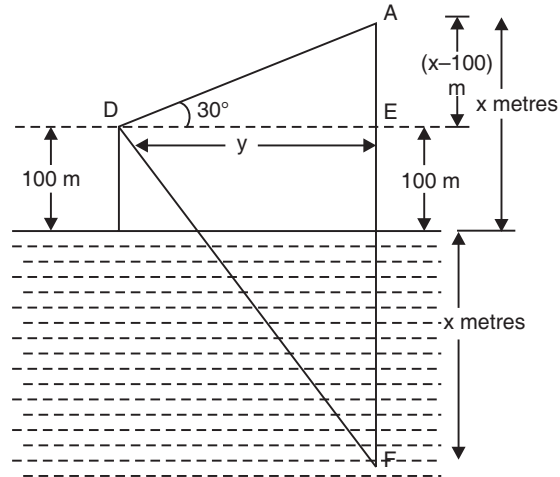
$$\text{In right } \triangle DEF, \tan 60^\circ = \frac{EF}{DE}$$

$$\begin{aligned} \Rightarrow \frac{EF}{DE} &= \sqrt{3} \\ \Rightarrow \frac{x+100}{y} &= \sqrt{3} \\ \Rightarrow \sqrt{3} y &= x + 100 \end{aligned}$$

$$\text{But } y = \sqrt{3} (x - 100)$$

$$\begin{aligned} \therefore \sqrt{3} \times \sqrt{3} (x - 100) &= x + 100 \\ \Rightarrow 3(x - 100) &= x + 100 \\ \Rightarrow 3x - 300 - x &= 100 \\ \Rightarrow 2x &= 100 + 300 \\ \Rightarrow 2x &= 400 \\ \Rightarrow x &= \frac{400}{2} = 200 \end{aligned}$$

Thus, the height of the stationary helicopter = **200 m**.



- Q. 7.** The angle of elevation of an aeroplane from a point on the ground is 60° . After a flight of 15 seconds, the angle of elevation changes to 30° . If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the aeroplane. (AI CBSE 2008 C)

Sol. In the figure, let E and C be the two locations of the aeroplane.

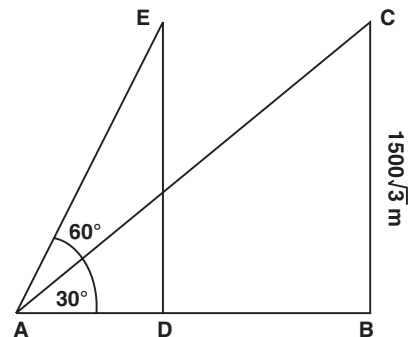
$$\text{Height } BC = ED$$

$$= 1500\sqrt{3} \text{ m}$$

In right $\triangle ABC$, we have:

$$\begin{aligned} \frac{BC}{AB} &= \tan 30^\circ = \frac{1}{\sqrt{3}} \\ \Rightarrow \frac{1500\sqrt{3}}{AB} &= \frac{1}{\sqrt{3}} \\ \therefore AB &= \sqrt{3} \times 1500 \times \sqrt{3} \text{ m} \\ &= 3 \times 1500 \text{ m} = 4500 \text{ m} \end{aligned}$$

In right $\triangle ADE$, we have:



$$\begin{aligned}\frac{ED}{AD} &= \tan 60^\circ = \sqrt{3} \\ \Rightarrow \frac{1500\sqrt{3}}{AD} &= \sqrt{3} & [\because ED = BC] \\ \Rightarrow AD &= \frac{1500\sqrt{3}}{\sqrt{3}} = 1500 \text{ m}\end{aligned}$$

Since the distance travelled in 15 seconds = $AB - AD$
 $= 4500 - 1500 = 3000 \text{ m}$

Since, $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$$\therefore \text{Speed of the aeroplane} = \frac{3000}{15} \text{ m/s} = 200 \text{ m/s.}$$

Q. 8. A spherical balloon of radius r subtends an angle θ at the eye of the observer. If the angle of elevation of its centre is ϕ , find the heights of centre of the balloon. [NCERT Exemplar]

Sol. In the figure, let O be the centre of the balloon, and A be the eye of the observer. r be the radius.

$$\therefore OP = r \text{ and } \angle PAQ = \theta$$

Also, $\angle OAB = \phi$

Let the height of the centre of the balloon be ' h ' $\Rightarrow OB = h$.

In $\triangle OAP$, $\angle OPA = 90^\circ$

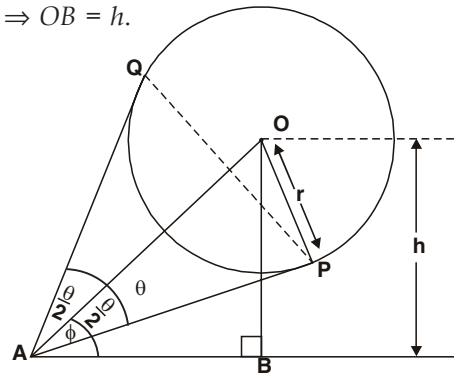
$$\Rightarrow \sin \frac{\theta}{2} = \frac{r}{s}, \text{ where } OA = s \dots(1)$$

$$\text{From, } \triangle OAB, \sin \phi = \frac{h}{s} \dots(2)$$

Now, from (1) and (2),

$$\begin{aligned}\frac{\sin \phi}{\sin \frac{\theta}{2}} &= \frac{\frac{h}{s}}{\frac{r}{s}} = \frac{h}{s} \times \frac{s}{r} = \frac{h}{r} \\ \Rightarrow h &= r \left[\frac{\sin \phi}{\sin \frac{\theta}{2}} \right]\end{aligned}$$

$$\Rightarrow h = r \cdot \sin \phi \cdot \operatorname{cosec} \frac{\theta}{2}$$



$$\therefore \frac{1}{\sin \frac{\theta}{2}} = \operatorname{cosec} \frac{\theta}{2}$$

Q. 9. As observed from the top of a light house, 100 m high above sea level, the angle of depression of a ship sailing directly towards it, changes from 30° to 60° . Determine the distances travelled by the ship during the period of observation. [Use $\sqrt{3} = 1.732$] (AI CBSE 2004)

Sol. Let A represents the position of the observer such that
 $AB = 100 \text{ m}$

∴ In right $\triangle ABC$, we have

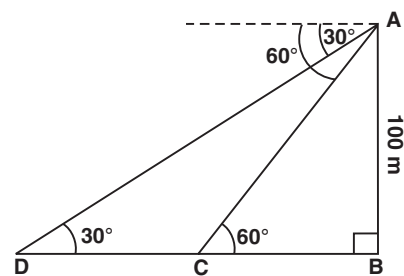
$$\begin{aligned}\frac{AB}{BC} &= \tan 60^\circ \\ \Rightarrow \frac{100}{BC} &= \sqrt{3} \Rightarrow \sqrt{3} BC = 100 \\ \Rightarrow BC &= \frac{100}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} \times \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3} \\ &= \frac{100 \times 1.732}{3} = 57.73 \text{ m}\end{aligned}$$

In right $\triangle ABD$, we have:

$$\begin{aligned}\frac{AB}{BD} &= \tan 30^\circ = \frac{1}{\sqrt{3}} \\ \Rightarrow \frac{100}{BD} &= \frac{1}{\sqrt{3}} \\ \Rightarrow BD &= \sqrt{3} \cdot 100 = 1.732 \times 100 \\ \Rightarrow BD &= 173.2 \text{ m}\end{aligned}$$

∴ The distance travelled

$$\begin{aligned}CD &= BD - BC \\ &= (173.2 - 57.73) \text{ m} = \mathbf{115.47 \text{ m}}\end{aligned}$$



- Q. 10.** The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60m high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them. (CBSE Delhi 2014)

Sol. Let AB is building = 60 m and DC is the tower

$$\begin{aligned}\text{In rt. } \triangle AED, \quad \frac{DE}{x} &= \tan 30^\circ = \frac{1}{\sqrt{3}} \\ \therefore x &= \sqrt{3} \times DE \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\text{In rt. } \triangle ABC, \quad \frac{AB}{BC} &= \tan 60^\circ = \sqrt{3} \\ \Rightarrow \frac{60}{x} &= \sqrt{3} \Rightarrow x = \frac{60}{\sqrt{3}} \quad \dots(2)\end{aligned}$$

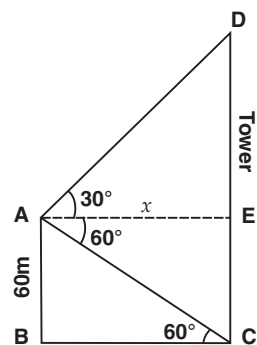
Substituting the value of x from (2) in (1), we have :

$$\sqrt{3} DE = \frac{60}{\sqrt{3}} \Rightarrow DE = \frac{60}{\sqrt{3} \times \sqrt{3}} = 20$$

⇒ Difference between the heights of building and tower = 20 m

Distance between the tower and building

$$= x = \sqrt{3} \times 20 = 1.732 \times 20 \text{ m} = 34.64 \text{ m}$$



TEST YOUR SKILLS

1. A person standing on the bank of a river observes that the angle of elevation of the top of a tower standing on the opposite bank is 60° . When he moves 40 m away from the bank, he finds the angle of elevation to be 30° . Find the height of the tower and the width of the river. [Use $\sqrt{3} = 1.732$] [CBSE 2008]

2. A straight highway leads to the foot of a tower. A man standing at top of the tower observes a car at angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point.
[AI CBSE 2008]
3. An aeroplane, when 3000 m high, passes vertically above another aeroplane at an instant, when the angle of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes.
[Use $\sqrt{3} = 1.732$] [CBSE 2011, 2012 CBSE 2008 F]
4. The angle of elevation of an aeroplane from a point A on the ground is 60° . After a flight of 30 seconds, the angle of elevation changes to 30° . If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed, in km/hr, of the plane. [CBSE 2008 F]
5. The angle of elevation of a jet fighter from a point A on the ground is 60° . After a flight of 10 seconds, the angle of elevation changes to 30° . If the jet is flying at a speed of 648 km/hr, find the constant height at which the jet is flying [Use $\sqrt{3} = 1.732$]
[CBSE 2012] [AI CBSE 2008]
6. The angle of elevation of a jet fighter from a point A on the ground is 60° . After a flight of 10 seconds, the angle of elevation changes to 30° . If the jet is flying at a speed of 432 km/hr, find the constant height at which the jet is flying [Use $\sqrt{3} = 1.732$]
[CBSE 2012] [AI CBSE 2008]
7. The angle of elevation of a jet fighter from a point A on the ground is 60° . After a flight of 15 seconds, the angle of elevation changes to 30° . If the jet is flying at a speed of 720 km/hr, find the constant height at which the jet is flying. [use $\sqrt{3} = 1.732$]
[AI CBSE 2008, 2014] [CBSE 2012]
8. A statue 1.46 m tall standing on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal. [Use $\sqrt{3} = 1.73$]
[CBSE 2008, 2012]
9. From the top of a house, h metres high from the ground, the angles of elevation and depression of the top and bottom of a tower on the other side of the street are θ and ϕ respectively. Prove that the height of the tower is $h(1 + \tan \theta \cot \phi)$. [AI CBSE, 2006, 2007]
10. A window in a building is at a height of 10 m from the ground. The angle of depression of a point P on the ground from the window is 30° . The angle of elevation of the top of the building from the point P is 60° . Find the height of the building. [AI CBSE 2007]
11. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point A on the ground is 60° and the angle of depression of point A from the top of the tower is 45° . Find the height of the tower. [Take $\sqrt{3} = 1.732$]
[AI CBSE 2004, 2007]
12. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30° . A girl standing on the roof of 20 m high building finds the angle of elevation of the same bird to be 45° . Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl. [CBSE 2007]

13. The angle of elevation of the top of a hill at the foot of the tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30° . If the tower is 50 m high, find the height of the hill. [AI CBSE 2006 C]
14. The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is 30° . On advancing 150 metres towards the foot of the tower, the angle of elevation becomes 60° . Show that the height of the tower is 129.9 metres. [Use $\sqrt{3} = 1.732$] [CBSE 2006 C]
15. From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of the opposite house is 23.66 metres. [Take $\sqrt{3} = 1.732$] [CBSE 2006 C]
16. A man standing on the deck of a ship, which 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30° . Calculate the distance of the hill from the ship and the height of the hill. [CBSE 2012] [AI CBSE 2006]
17. From a point 'A' on a straight road the angle of elevation of the top of a vertical tower situated on the roof of a vertical building on the same road is θ . The angle of elevation of the bottom of the tower from a point B on the road is again θ . The height of the building is 50 m. If $AB : BY$ is 2 : 5, where Y is the base of building, then show that the height of the tower is 20 m.
18. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30° . Find the distance between the two towers and also the height of the other tower. [NCERT Exemplar]
19. An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer. [NCERT Exemplar]
20. From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° . Find the height of the tower.
21. The angle of elevation of a cloud from a point 200 m, above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60° . Find the height of the cloud. [CBSE 2011, 2012]
22. A tree 12 m high is broken by the wind in such a way that its top touches the ground and makes an angle of 60° with the ground. At what height from the bottom the tree is broken by the wind? [CBSE 2011]
23. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° . If the height of the lighthouse is 200m, find the distance between the two ships. [Use $\sqrt{3} = 1.732$] [CBSE 2014]

Hint:

In rt $\triangle AMP$, we have:

$$\frac{PM}{AM} = \tan 60^\circ \Rightarrow \frac{200}{x} = \sqrt{3}$$

$$\Rightarrow x = \frac{200}{\sqrt{3}} \quad \dots(1)$$

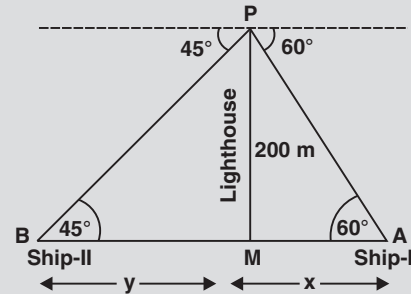
In rt ΔBMP , we have :

$$\frac{PM}{BM} = \tan 45^\circ \Rightarrow \frac{200}{y} = 1$$

$$\Rightarrow y = 200 \quad \dots(2)$$

Solving (1) and (2),

we get required distance $(x + y) = 315.4 \text{ m}$



24. Two ships are approaching a lighthouse from opposite directions. The angles of depression of the two ships from the top of the lighthouse are 30° and 45° . If the distance between the two ships is 100 m, find the height of the lighthouse. [Use $\sqrt{3} = 1.732$]

[AI. CBSE (Foreign) 2014]

25. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45° . If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60° , then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

[AI. CBSE 2014]

ANSWERS

TEST YOUR SKILLS

- | | | | | |
|--------------------|--------------------------|------------|--------------------------|--------------------------|
| 1. 34.64 m; 20 m | 2. 3 seconds | 3. 1268 m | 4. 864 km/h | 5. 1558.8 m |
| 6. 1039.2 m | 7. 2598 m | 8. 2 m | 10. 30 m | 11. 6.82 m |
| 12. $30\sqrt{2}$ m | 13. 150 m | 15. 23.6 m | 16. 40 m; $10\sqrt{2}$ m | 18. $10\sqrt{3}$ m, 10 m |
| 19. 45° | 20. $20(\sqrt{3} - 1)$ m | 21. 400 m | 22. 5.569 m. | 23. 315.4 m |
| 24. 36.6 m | 25. 87.6 m | | | |