KINETICS

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IIT-JEE | Medical | Foundations

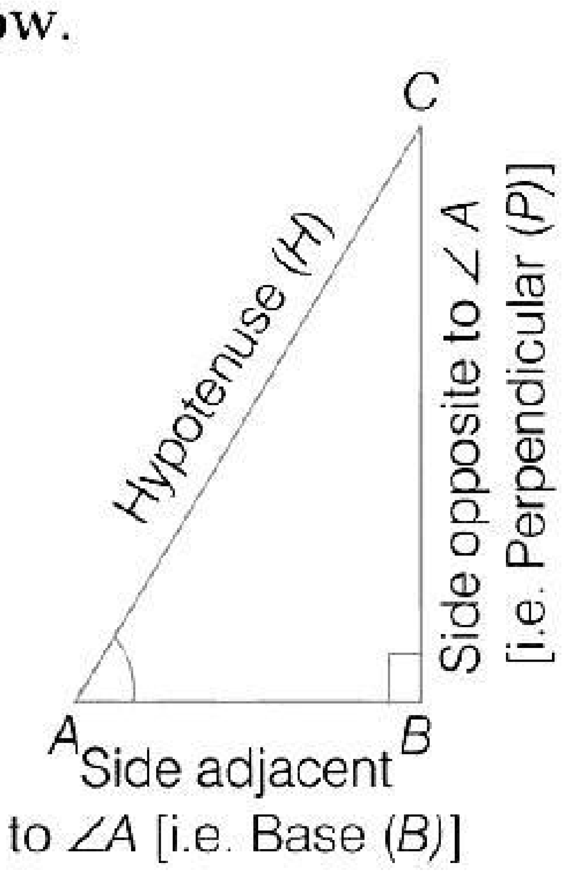
Introduction to Trigonometry

Quick Revision

Trigonometric Ratios

The ratios of the sides of a right angled triangle with respect to its acute angles, are called trigonometric ratios.

Trigonometric ratios are also called T-ratio. Trigonometric ratios of $\angle A$ in right angled ΔABC are defined below.



(i) sine A or $\sin A = \dfrac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} \left(\text{i.e. } \dfrac{P}{H}\right)$

$= \dfrac{BC}{AC}$

(ii) cosine A or $\cos A$

$= \dfrac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} \left(\text{i.e. } \dfrac{B}{H}\right) = \dfrac{AB}{AC}$

(iii) tangent A or $\tan A$

$= \dfrac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} \left(\text{i.e. } \dfrac{P}{B}\right) = \dfrac{BC}{AB}$

(iv) cosecant A or cosec A

$= \dfrac{\text{Hypotenuse}}{\text{Side opposite to } \angle A} \left(\text{i.e. } \dfrac{H}{P}\right) = \dfrac{AC}{BC}$

(v) secant A or sec A

$= \dfrac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} \left(\text{i.e. } \dfrac{H}{B}\right) = \dfrac{AC}{AB}$

(vi) cotangent A or cot A

$= \dfrac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A} \left(\text{i.e. } \dfrac{B}{P}\right) = \dfrac{AB}{BC}$

Similarly trigonometric ratios of $\angle C$ are

(a) $\sin C = \dfrac{AB}{AC}$

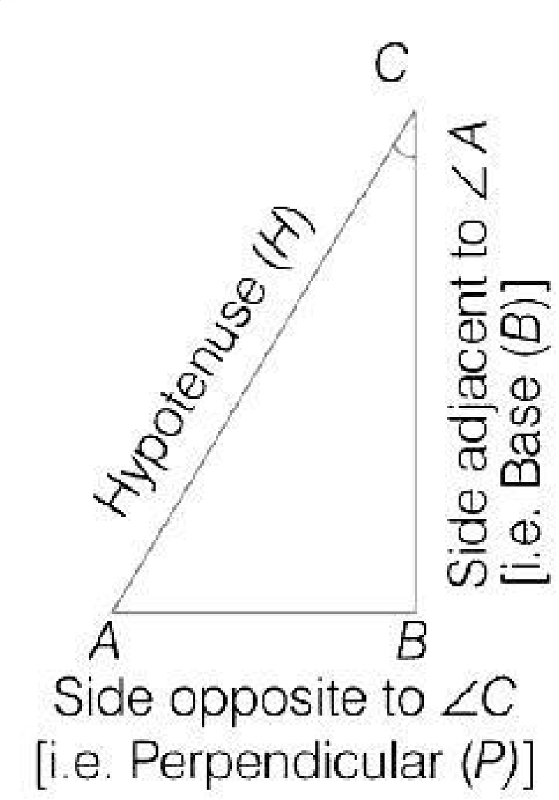
(b) $\cos C = \dfrac{BC}{AC}$

(c) $\tan C = \dfrac{AB}{BC}$

(d) $\text{cosec}\, C = \dfrac{AC}{AB}$

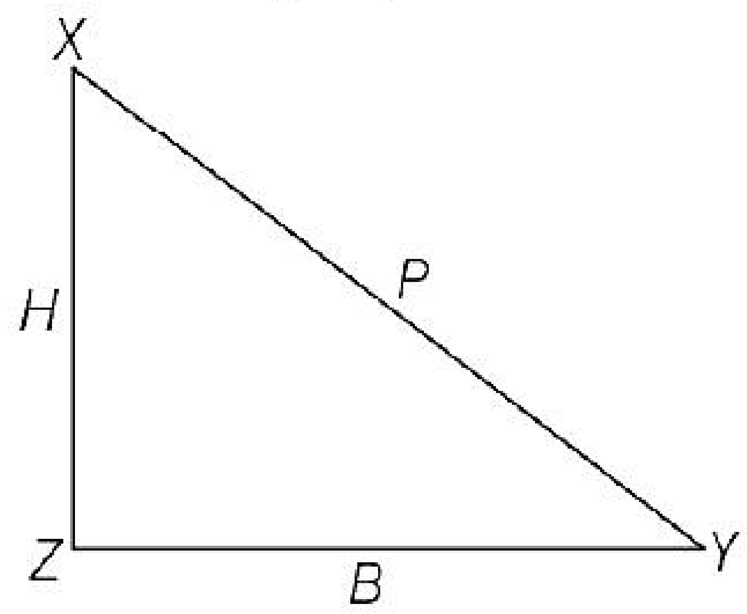
(e) $\sec C = \dfrac{AC}{BC}$

(f) $\cot C = \dfrac{BC}{AB}$



A Popular Technique to Remember T-ratios i.e. $\frac{PBP}{HHB}$

Pandit (P)	Badari (B)	Prasad (P)
Har (H)	Har (H)	Bholay (B)



Then, $\sin\theta = \frac{P}{H}$, $\cos\theta = \frac{B}{H}$, $\tan\theta = \frac{P}{B}$

$\Rightarrow$ $\text{cosec}\,\theta = \frac{H}{P}$, $\sec\theta = \frac{H}{B}$, $\cot\theta = \frac{B}{P}$

where, P is perpendicular, B is base and H is hypotenuse.

Important Points

(i) In an isosceles right ΔABC, right angled at B, the trigonometric ratios obtained by taking either $\angle A$ or $\angle C$, both give the same value.

(ii) The value of each of the trigonometric ratios of an angle does not depend on the size of the triangle. It only depends on the angle.

(iii) It is clear that the values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

(iv) If one of the trigonometric ratios of an acute angle is known, then remaining trigonometric ratios of that angle can be determined easily.

(v) Each trigonometric ratio is a real number and has no unit.

(vi) As, the hypotenuse is the longest side in a right angled triangle, the value of $\sin A$ or $\cos A$ is always less than 1 (or in particular equal to 1) whereas the value of $\sec A$ or $\text{cosec}\,A$ is always greater than or equal to 1.

Relation Between Trigonometric Ratios

(i) $\sin A = \frac{1}{\text{cosec}\,A}$, $\text{cosec}\,A = \frac{1}{\sin A}$

(ii) $\cos A = \frac{1}{\sec A}$, $\sec A = \frac{1}{\cos A}$

(iii) $\tan A = \frac{1}{\cot A}$, $\cot A = \frac{1}{\tan A}$

(iv) $\tan A = \frac{\sin A}{\cos A}$

(v) $\cot A = \frac{\cos A}{\sin A}$

Values of Trigonometric Ratios for Some Specific Angles

Angles	**0°**	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Here, ∞ = undefined

Important Points

(i) The value of $\sin\theta$ increase from 0 to 1 and $\cos\theta$ decrease from 1 to 0, where $0 \le \theta \le 90°$.

(ii) In the case of $\tan\theta$, the values increase from 0 to ∞, where $0 \le \theta \le 90°$.

(iii) In the case of $\cot\theta$, the values decrease from ∞ to 0, where $0 \le \theta \le 90°$.

(iv) In the case of $\text{cosec}\,\theta$, the values decrease from ∞ to 1, where $0 \le \theta \le 90°$.

(v) In the case of $\sec\theta$, the values increase from 1 to ∞, where $0 \le \theta \le 90°$.

(vi) Division by 0 is not allowed, since 1/0 is indeterminate (not defined).

Trigonometric Identity

An equation is called an identity when it is true for all values of the variables involved. Similarly, an equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle involved. For any acute angle θ, we have

(i) $\sin^2\theta + \cos^2\theta = 1$ or $\sin^2\theta = 1 - \cos^2\theta$
or $\cos^2\theta = 1 - \sin^2\theta$

(ii) $\sec^2\theta - \tan^2\theta = 1$ or $1 + \tan^2\theta = \sec^2\theta$
or $\sec^2\theta - 1 = \tan^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$ or $\text{cosec}^2\theta - \cot^2\theta = 1$
or $\cot^2\theta = \text{cosec}^2\theta - 1$

Note $\sin^2\theta = (\sin\theta)^2$ but $\sin\theta^2 \neq (\sin\theta)^2$. The same is true for all other trigonometric ratios.

Representation of a Trigonometric Ratio in Terms of Any Other Trigonometric Ratio

	sin θ	**cos θ**	**tan θ**	**cot θ**	**sec θ**	**cosec θ**
sin θ	$\sin\theta$	$\sqrt{(1-\cos^2\theta)}$	$\frac{\tan\theta}{\sqrt{(1+\tan^2\theta)}}$	$\frac{1}{\sqrt{(1+\cot^2\theta)}}$	$\frac{\sqrt{(\sec^2\theta-1)}}{\sec\theta}$	$\frac{1}{\text{cosec}\,\theta}$
cos θ	$\sqrt{(1-\sin^2\theta)}$	$\cos\theta$	$\frac{1}{\sqrt{(1+\tan^2\theta)}}$	$\frac{\cot\theta}{\sqrt{(1+\cot^2\theta)}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{(\text{cosec}^2\theta-1)}}{\text{cosec}\,\theta}$
tan θ	$\frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}$	$\frac{\sqrt{(1-\cos^2\theta)}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{(\sec^2\theta-1)}$	$\frac{1}{\sqrt{(\text{cosec}^2\theta-1)}}$
cot θ	$\frac{\sqrt{(1-\sin^2\theta)}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{(1-\cos^2\theta)}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{(\sec^2\theta-1)}}$	$\sqrt{\text{cosec}^2\theta-1}$
sec θ	$\frac{1}{\sqrt{(1-\sin^2\theta)}}$	$\frac{1}{\cos\theta}$	$\sqrt{(1+\tan^2\theta)}$	$\frac{\sqrt{(1+\cot^2\theta)}}{\cot\theta}$	$\sec\theta$	$\frac{\text{cosec}\,\theta}{\sqrt{(\text{cosec}^2\theta-1)}}$
cosec θ	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{(1-\cos^2\theta)}}$	$\frac{\sqrt{(1+\tan^2\theta)}}{\tan\theta}$	$\sqrt{(1+\cot^2\theta)}$	$\frac{\sec\theta}{\sqrt{(\sec^2\theta-1)}}$	$\text{cosec}\,\theta$

Objective Questions

Multiple Choice Questions

1. Trigonometry is branch of Mathematics in which we deal with the relationship between angle and sides of a triangle.

(a) True (b) False
(c) Cannot say (d) Partially True/False

2. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is ***[NCERT Exemplar]***

(a) 3/5 (b) 3/4
(c) 4/3 (d) 5/3

3. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to ***[NCERT Exemplar]***

(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

4. If $\cos\theta = \frac{2}{3}$, then the value of $2\sec^2\theta + 2\tan^2\theta - 9$ is

(a) 2 (b) −2
(c) 1 (d) 3

5. If $x \cos A = 8$ and $15 \operatorname{cosec} A = 8 \sec A$, then the value of x is

(a) 20 (b) 16
(c) 17 (d) 13

6. If $\alpha \sin A = 5$ and $7 \operatorname{cosec} A = 6 \sec A$, then the value of α is

(a) $\frac{46}{45}$ (b) $\frac{46}{7}$
(c) $\frac{49}{46}$ (d) None of these

7. If θ is an acute angle and $4\sin\theta = 3$, then the value of $4\sin^2\theta - 3\cos^2\theta + 2$ is

(a) $\frac{45}{16}$ (b) $\frac{35}{18}$
(c) $\frac{32}{16}$ (d) $\frac{47}{16}$

8. If $\sin A = \frac{4}{5}$, then the value of $\frac{1-\sin A}{1+\cos A}$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$
(c) 2 (d) $\frac{3}{5}$

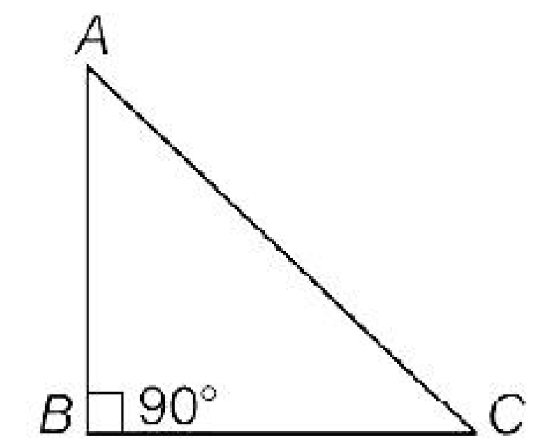
9. In a ΔPQR, $\angle Q = 90°$. If $PQ = 10$ cm and $PR = 15$ cm. Then, the value of $\tan^2 P + \sec^2 P + 1$ is

(a) $\frac{5}{2}$ (b) $\frac{3}{17}$
(c) $\frac{9}{2}$ (d) $\frac{4}{5}$

10. In ΔABC, if $AB = BC$, $\angle B = 90°$, then the value $\sin A$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
(c) 0 (d) $\frac{1}{2}$

11. In ΔABC, if $AB = 2BC$, $\angle B = 90°$, then the value of $\sec A$ is



(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{\sqrt{5}}{2}$ (d) $\frac{1}{2}$

12. The value of $\cos\theta$ increase as θ increase.

(a) True (b) False
(c) Cannot say (d) Partially True/False

13. Is $\tan^2 30° + \cot^2 30° = \frac{10}{3}$?

(a) True (b) False
(c) Cannot say (d) Partially True/False

14. $\sin 2A = 2\sin A$ is true when A is equal to

(a) 0° (b) 30°
(c) 45° (d) 60°

15. $\cot^2 30° + \operatorname{cosec} 30° + 3\tan^2 30°$ is equal to

(a) 3 (b) 1
(c) 0 (d) 6

16. The value of $\frac{2\tan 30°}{1-\tan^2 30°}$ will be

(a) cos 60° (b) sin 60°
(c) tan 60° (d) cot 60°

17. If $\tan\theta + \frac{1}{\tan\theta} = 2$, then the value of $\operatorname{cosec}\theta$ is

(a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$

18. If $x \tan 45° \cos 60° = \sin 60° \cot 60°$, then the value of x will be

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

19. $7\cos 30°+5\tan 30°+6\cot 60°$ is

(a) $\frac{43}{2\sqrt{3}}$ (b) $\frac{41\sqrt{3}}{2}$
(c) $\frac{47}{2\sqrt{3}}$ (d) $\frac{49\sqrt{3}}{2}$

20. If $\tan\theta - \frac{4}{\tan\theta} = 3$, then $\sin^2\theta$ is

(a) $\frac{4}{17}$ (b) $\frac{3}{17}$
(c) $\frac{16}{17}$ (d) $\frac{5}{17}$

21. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(a) 60° (b) 90°
(c) 30° (d) 45°

22. If $\tan A = \frac{1}{\sqrt{3}}$ and $\tan B = \sqrt{3}$, then $\tan(A + B)$ is

(a) 0 (b) $\frac{1}{\sqrt{3}}$
(c) 1 (d) ∞

23. If $\sqrt{3}\tan\theta = 2\sin\theta$, then the value $\sin^2\theta - \cos^2\theta$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

24. If $\sin\theta - \cos\theta = 0$, then the value of $\sin^4\theta + \cos^4\theta$ will be ***[NCERT Exemplar]***

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) 1

25. If $m\tan 30°\cot 60° = \sin 45°\cos 45°$, then the value of m will be

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$
(c) $\frac{1}{3}$ (d) $\frac{3}{4}$

26. The value of $\frac{2\tan 30°}{1 + \tan^2 30°}$ is

(a) sin60° (b) cos60°
(c) tan60° (d) sin30°

27. The value of $\frac{1 - \tan^2 45°}{1 + \tan^2 45°}$ is

(a) tan90° (b) 1
(c) sin45° (d) 0

28. The value of $\frac{\tan 30°}{\cot 60°}$ is

(a) 1 (b) −1
(c) 2 (d) $\frac{1}{2}$

29. If A, B, C are the angles of a ΔABC, then the value of $\tan\left(\frac{B+C}{2}\right)$ is

(a) $\cot\frac{B}{2}$ (b) $\tan\frac{A}{2}$
(c) $\cot\frac{C}{2}$ (d) $\cot\frac{A}{2}$

30. If $\cos A + \cos^2 A = 1$, then the value of $\sin^2 A + \sin^4 A$ is

(a) 0 (b) 1
(c) −1 (d) 2

31. Is $\sin(A + B) = \sin A + \sin B$?

(a) True
(b) False
(c) Cannot say
(d) Partially True/False

32. Is $\frac{1 - \sin\theta}{1 + \sin\theta} = \sec^2\theta - \tan^2\theta$?

(a) True
(b) False
(c) Cannot say
(d) Partially True/False

33. The value of $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ is

(a) $\sec^2 A$
(b) −1
(c) $\cot^2 A$
(d) $\tan^2 A$

34. The value of $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \csc\theta)$ is

(a) 0 (b) 1
(c) 2 (d) −1

35. If $4x = \text{cosec}\,\theta$ and $\frac{4}{x} = \cot\theta$, then the value of $4\left[x^2 - \frac{1}{x^2}\right]$ is

(a) $\frac{1}{4}$ (b) 4
(c) 2 (d) $\frac{1}{2}$

36. The value of $\frac{\sin\theta\tan\theta}{1-\cos\theta} + \tan^2\theta - \sec^2\theta$ is

(a) $\sin\theta\cos\theta$ (b) $\sec\theta$
(c) $\tan\theta$ (d) $\text{cosec}\,\theta$

37. If $\sec\theta = x + \frac{1}{4x}$, then the value of $\sec\theta + \tan\theta$ is

(a) $4x$ (b) $2x$
(c) x (d) $\frac{x}{2}$

38. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then the value of $\tan^2\theta + \cot^2\theta$ is

(a) $\frac{3}{10}$ (b) $\frac{-10}{3}$
(c) $\frac{10}{3}$ (d) $\frac{-20}{3}$

39. If $\tan\theta + \sec\theta = l$, then the value of $\sec\theta$ is

(a) $\frac{l^2-1}{2l}$ (b) $\frac{l^2+1}{2l}$
(c) $\frac{l^2-1}{l}$ (d) $\frac{2(l^2-1)}{l}$

40. If $\tan A = a\tan B$ and $\sin A = b\sin B$, then the value of $\cos^2 A$ is

(a) $\frac{a^2-1}{b^2-1}$ (b) $\frac{a^2+1}{b^2-1}$
(c) $\frac{b^2-1}{a^2+1}$ (d) $\frac{b^2-1}{a^2-1}$

41. If $\text{cosec}\,A - \cot A = q$, then the value of $\frac{q^2-1}{q^2+1} + \cos A$ is

(a) 1 (b) 0
(c) -1 (d) 2

42. If $x = r\sin A\cos B$, $y = r\sin A\sin B$ and $z = r\cos A$, then the value of $x^2 + y^2 + z^2$ is

(a) $\frac{r^2}{2}$ (b) r^2
(c) $r^2 - 1$ (d) $r^2 + 1$

43. If $\sin^2 A = 1$, then the value of $\sin^2 A - \cos^2 A$ is

(a) 1 (b) 0
(c) 2 (d) None of these

44. Match the following.

	List I		List II
P.	$\frac{\sin 0°}{\cos 90°} + \sin 45°$	1.	$\left(\frac{1-\sqrt{3}}{2}\right)$
Q.	$\cos 60° - \sin 60°$	2.	$1 + \frac{\sqrt{2}}{3}$
R.	$\sec 30°\sin 60° + \cos 45°\,\text{cosec}\,60°$	3.	1
S.	$\frac{\cos^3 30° - \cos^3 60°}{\sin^3 60° - \sin^3 30°}$	4.	$\frac{1}{\sqrt{2}}$

	P	Q	R	S
(a)	2	4	3	1
(b)	3	1	4	2
(c)	2	3	4	1
(d)	4	1	2	3

45. Match the following.

	List I		List II
P.	$1 + \frac{\cot^2\theta}{1+\text{cosec}\,\theta}$	1.	$2\tan\theta$
Q.	$\frac{\cos\theta}{\text{cosec}\,\theta + 1} + \frac{\cos\theta}{\text{cosec}\,\theta - 1}$	2.	$\left(\frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}\right)^2$
R.	$\tan^2\theta + \cot^2\theta - 2$	3.	$(\text{cosec}\,\theta - \cot\theta)^2$
S.	$\frac{1-\cos\theta}{1+\cos\theta}$	4.	$\sec\theta\cot\theta$

	P	Q	R	S
(a)	3	4	2	1
(b)	3	1	4	2
(c)	2	3	4	1
(d)	4	1	2	3

Assertion-Reasoning MCQs

Directions (Q. Nos. 46-55) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is false.
(d) A is false; R is true.

46. Assertion $2\cos\theta = a + \frac{1}{a}$, where $a > 0,\ a \neq 1$

Reason $-1 \le \cos\theta \le 1$ for all values of θ.

47. Assertion The value of each of the trigonometric ratios of an angle does not depend on the size of the triangle. It only depends on the angle.

Reason In right ΔABC, $\angle B = 90°$ and $\angle A = \theta°$ $\sin\theta = \frac{BC}{AC} < 1$ and $\cos\theta = \frac{AB}{AC} < 1$ as hypotenuse is the longest side.

48. Assertion The equation $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is only possible when $x = y$.

Reason $\sec^2\theta > 1$ and therefore $(x-y)^2 < 0$.

49. Assertion In a right angled triangle, if $\tan\theta = \frac{3}{4}$, then greatest side of the triangle is 5 units.

Reason $(\text{greatest side})^2 = (\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

50. Assertion $\sin 60° = \cos 30°$.

Reason $\sin 2\theta = \sin\theta + \sin\theta$, where θ is an acute angle.

51. Assertion $\cos 60° - \sin 60°$ is negative.

Reason $\sin^2\theta - \cos^2\theta$ is positive, where θ is an acute angle.

52. Assertion $\frac{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} = -1$

Reason $\sin^2\theta + \cos^2\theta = -1$

53. Assertion $\cos^2 A - \sin^2 A = 1$, $\tan^2 A - \sec^2 A = 1$ are trigonometric identities.

Reason An equation involving trigonometric ratios of an angle is called a trigonometric identity. It is true for all values of the angles involved.

54. Assertion $(\cot\theta + 3)(3\cot\theta + 1) = 3\text{cosec}^2\theta + 10\cot\theta$

Reason $1 + \cot^2\theta = \text{cosec}^2\theta$

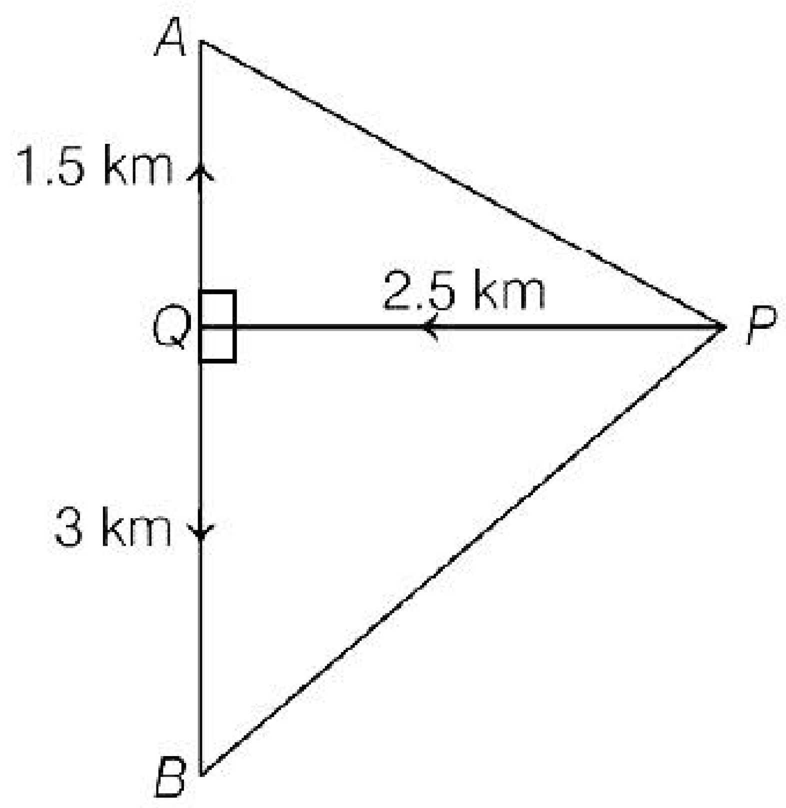
55. Assertion If $\sec\theta + \tan\theta = x$, then the value of $\sin\theta = \frac{x^2 - 1}{x^2 + 1}$

Reason $x + \frac{1}{x} = 2\tan\theta$ and $x - \frac{1}{x} = 2\sec\theta$.

Case Based MCQs

56. Two jet plane leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400 km/h and 500 km/h respectively.

Considering PQ as runway and A and B are any two points in the path followed by two jet planes, then answer the following questions.



(i) Find $\tan\theta$; if $\angle APQ = \theta$.

(a) $\frac{1}{5}$ (b) $\frac{2}{3\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{5}$

(ii) Find $\cot B$.

(a) $\frac{3}{4}$ (b) $\frac{6}{5}$

(c) $\frac{5}{6}$ (d) $\frac{15}{8}$

(iii) Find $\tan A$.

(a) 2/3 (b) $\sqrt{2}$

(c) $\frac{5}{3}$ (d) $\frac{2}{\sqrt{3}}$

(iv) Find $\sec A$.

(a) 1 (b) 2

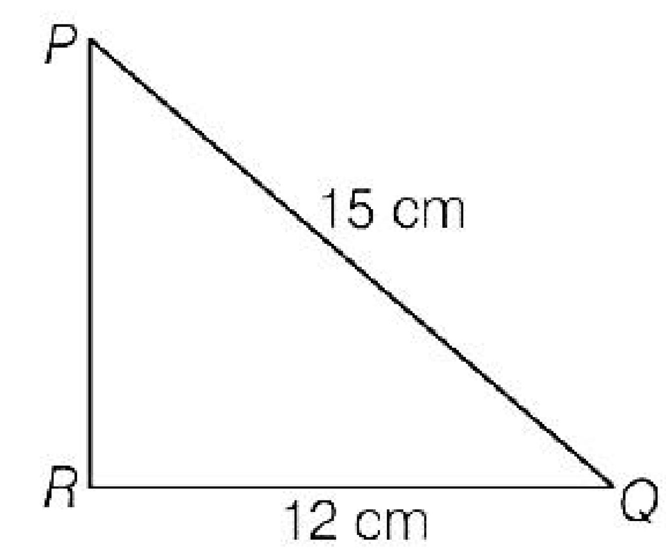
(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

(v) Find $\text{cosec}\, B$.

(a) $\frac{17}{8}$ (b) $\frac{8}{5}$

(c) $\frac{6}{5}$ (d) $\frac{8}{17}$

57. Kavita a student of class 10th, has to made a project on 'Introduction to Trigonometry'. She decides to make a Dog house which is triangular in shape. She uses cardboard to make the Dog house as shown in the figure. Considering the front side of Dog house as right angled triangle PQR, right angled at R, answer the following questions.



(i) If $\angle PQR = \theta$, then $\cos\theta$ is equal to

(a) $\frac{4}{5}$ (b) $\frac{5}{12}$

(c) $\frac{3}{5}$ (d) $\frac{13}{12}$

(ii) The value of $\sec\theta$ is

(a) $\frac{5}{12}$ (b) $\frac{5}{4}$

(c) $\frac{13}{12}$ (d) $\frac{5}{3}$

(iii) The value of $\frac{\tan\theta}{1+\tan^2\theta}$ is

(a) $\frac{5}{12}$ (b) $\frac{12}{25}$

(c) $\frac{60}{169}$ (d) $\frac{169}{60}$

(iv) The value of $\cot^2\theta - \text{cosec}^2\theta$ is

(a) −1 (b) 0

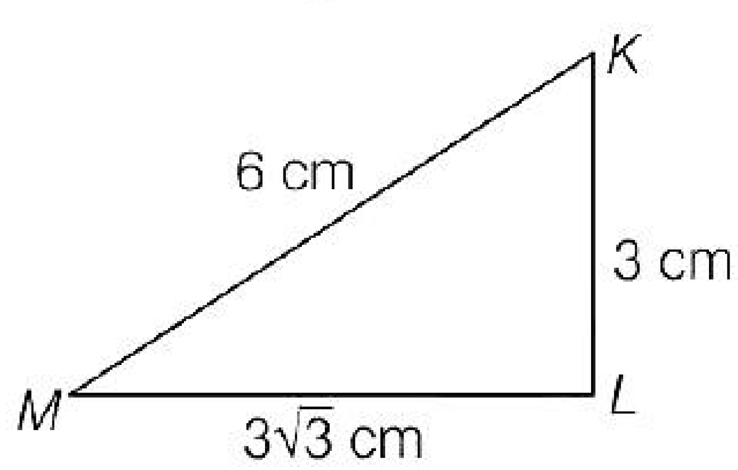
(c) 1 (d) 2

(v) The value of $\sin^2\theta + \cos^2\theta$ is

(a) 0 (b) 1

(c) −1 (d) 2

58. Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the bag and found some chips packed. She decided to eat. She open the chips packet and found that it forms a right angled triangle, with sides 3 cm, $3\sqrt{3}$ cm and 6 cm.



On the basis of above information, answer the following questions.

(i) The value of $\angle M$ =

(a) 30° (b) 60°

(c) 45° (d) None of these

(ii) The value of $\angle K$ =

(a) 45°

(b) 30°

(c) 60°

(d) None of the above

(iii) Find the value of $\tan M$.

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) 1

(d) None of the above

(iv) $\sec^2 M - 1 =$

(a) $\tan M$ (b) $\tan 2M$

(c) $\tan^2 M$ (d) None of these

(v) The value of $\frac{\tan^2 45° - 1}{\tan^2 45° + 1}$ is

(a) 0 (b) 1

(c) 2 (d) −1

59. Janvi and her father go to meet her friend Sanvi for a candle light event. When they reached to church. Janvi saw the top of the church which is triangular in shape. If she imagined the dimensions of the top as given in the figure, then answer the following questions.

(i) If D is the mid point of AC, then BD is

(a) 2m (b) 3m

(c) 4 m (d) 6 m

(ii) Measure of $\angle A$ is

(a) 30° (b) 60°

(c) 45° (d) None of these

(iii) Measure of $\angle C$ is

(a) 30° (b) 60°

(c) 45° (d) None of these

(iv) The value of $\sin A + \cos C$.

(a) 0 (b) 1

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

(v) The value of $\tan^2 C + \tan^2 A$.

(a) 0 (b) 1

(c) 2 (d) $\frac{1}{2}$

ANSWERS

Multiple Choice Questions

1. (a)	2. (b)	3. (c)	4. (b)	5. (c)	6. (b)	7. (d)	8. (a)	9. (c)	10. (a)
11. (c)	12. (b)	13. (a)	14. (a)	15. (d)	16. (c)	17. (c)	18. (a)	19. (a)	20. (c)
21. (b)	22. (d)	23. (b)	24. (b)	25. (b)	26. (a)	27. (d)	28. (a)	29. (d)	30. (b)
31. (b)	32. (b)	33. (d)	34. (c)	35. (a)	36. (b)	37. (b)	38. (c)	39. (b)	40. (d)
41. (b)	42. (b)	43. (a)	44. (d)	45. (d)					

Assertion and Reason

46. (d)	47. (b)	48. (a)	49. (a)	50. (c)	51. (c)	52. (c)	53. (d)	54. (a)	55. (c)

Case Study Based

56. (i) - (d); (ii) - (b); (iii) - (c); (iv) - (b); (v) - (b)
57. (i) - (a); (ii) - (b); (iii) - (b); (iv) - (a); (v) - (b)
58. (i) - (a); (ii) - (c); (iii) - (b); (iv) - (c); (v) - (a)
59. (i) - (c); (ii) - (c); (iii) - (c); (iv) - (d); (v) - (c)

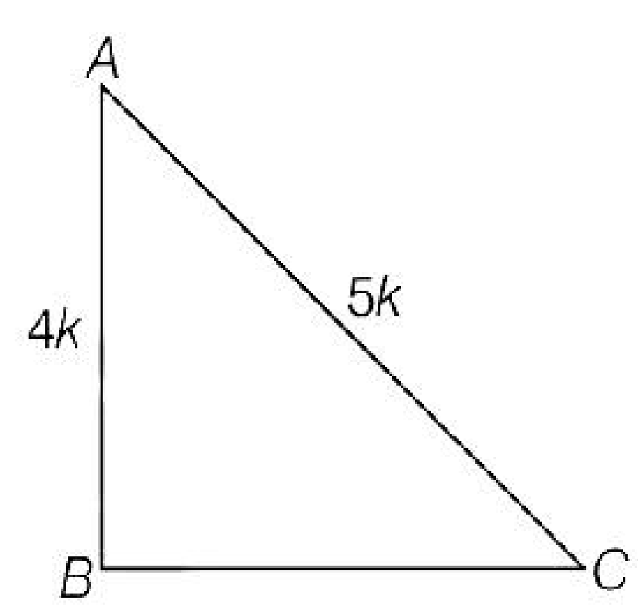
SOLUTIONS

1. (True) Trigonometry is branch of Mathematics in which we deal with the relationship between angle and sides of a triangle.

$$\therefore \quad \sin\theta = \frac{p}{h} \qquad \tan\theta = \frac{p}{b}$$

$$\operatorname{cosec}\theta = \frac{h}{p} \quad \sec\theta = \frac{h}{b} \quad \cot\theta = \frac{b}{p}$$

2. Given,



$$\cos A = \frac{4}{5}$$

$\therefore$ Let $AB = 4k$ and $AC = 5k$

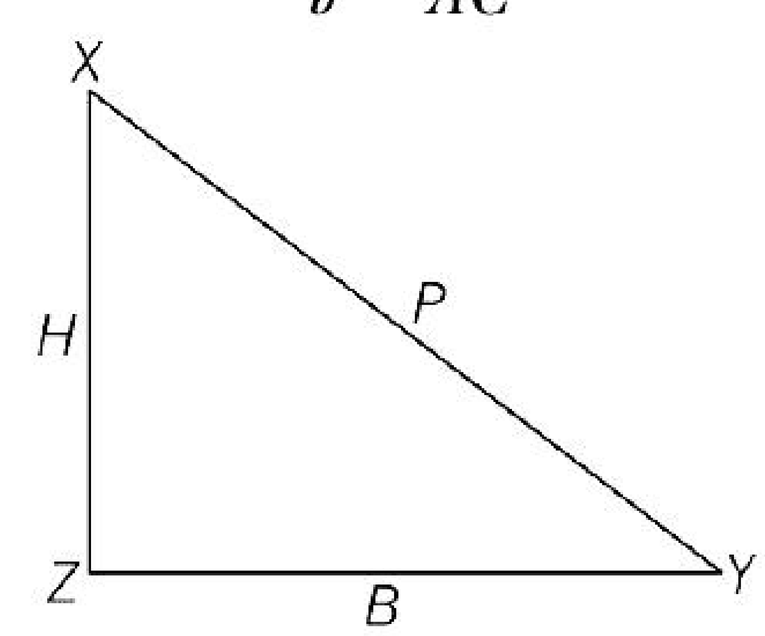
Apply Pythagoras Theorem

$$AB^2 + BC^2 = AC^2$$

$$BC = \sqrt{25k^2 - 16k^2} = 3k$$

Now, $\quad \tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

3. Given, $\quad \sin\theta = \frac{a}{b} = \frac{AB}{AC}$



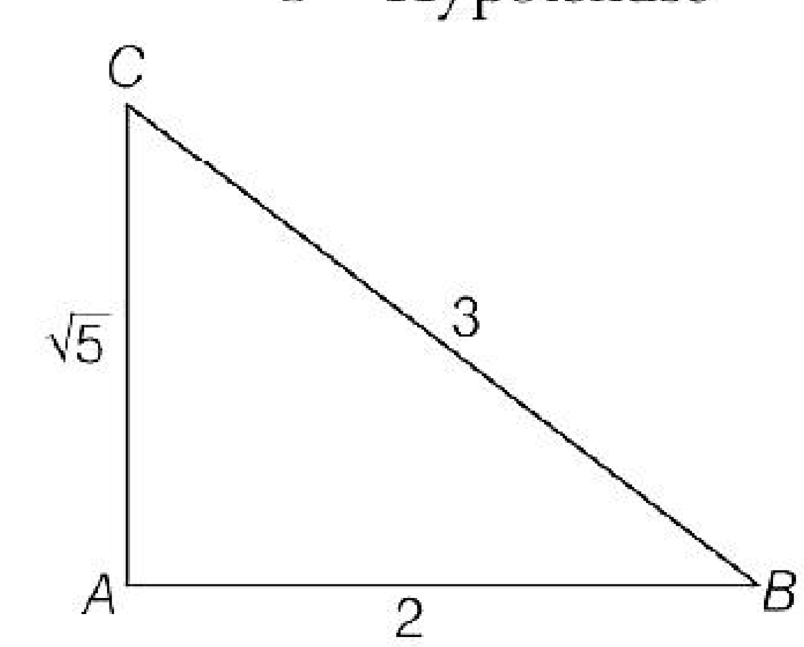
Let $AB = ak$, $AC = bk$

$$AB^2 + BC^2 = AC^2$$

[from Pythagoras theorem]

$$BC = \sqrt{b^2 - a^2} \Rightarrow \cos\theta = \frac{\sqrt{b^2 - a^2}}{b}$$

4. Given, $\cos\theta = \frac{2}{3} = \frac{\text{Base}}{\text{Hypotenuse}}$



Let $AB = 2k$ and $BC = 3k$

Apply Pythagoras theorem,

$$AB^2 + AC^2 = BC^2$$

$$\Rightarrow \quad (2k)^2 = AC^2 + (3k)^2$$

$$\Rightarrow \quad AC^2 = 9k^2 - 4k^2$$

$$\Rightarrow \quad AC = \sqrt{5}k$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{3}{2}, \tan\theta = \frac{p}{b} = \frac{\sqrt{5}}{2}$$

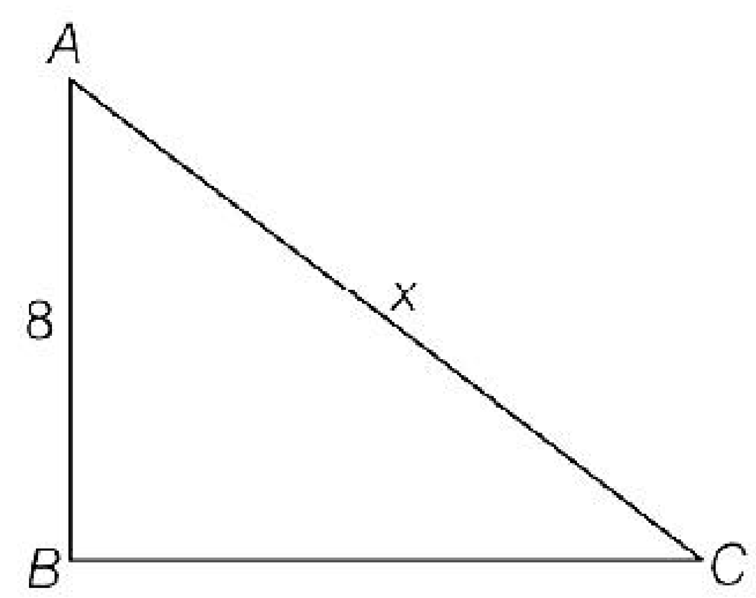
$2\sec^2\theta + 2\tan^2\theta - 9$

$$= 2\times\left(\frac{3}{2}\right)^2 + 2\times\left(\frac{\sqrt{5}}{2}\right)^2 - 9$$

$$= 2\times\frac{9}{4} + \frac{2\times 5}{4} - 9 = \frac{18 + 10 - 36}{4}$$

$$= \frac{28 - 36}{4} = -\frac{8}{4} = -2$$

5. Given, $x\cos A = 8$



$$\cos A = \frac{8}{x}$$

and $15\operatorname{cosec} A = 8\sec A$

$$\cos A = \frac{8}{x}$$

$$= \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

Let $AB = 8k$, $AC = xk$

Apply Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \quad (8k)^2 + BC^2 = (xk)^2$$

$$\Rightarrow \quad BC = (\sqrt{x^2 - 8^2})\,k$$

$$\Rightarrow \quad 15\operatorname{cosec} A = 8\sec A$$

$$\Rightarrow \quad \frac{15\times xk}{k\sqrt{x^2 - 8^2}} = \frac{8xk}{8k}$$

$$\Rightarrow \quad \sqrt{x^2 - 8^2} = 15$$

On squaring both side, we get

$$x^2 - 8^2 = (15)^2$$

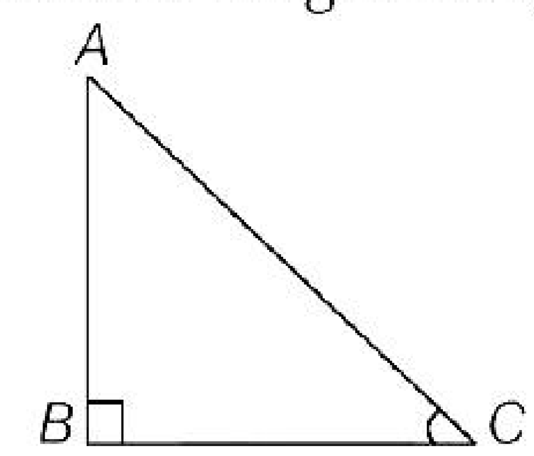
$$\Rightarrow \quad x^2 = 225 + 64$$

$$\Rightarrow \quad x^2 = 289$$

$$\Rightarrow \quad x = 17$$

6. $\sin A = \frac{5}{\alpha}$ and $7\operatorname{cosec}A = 6\sec A$

Let the ΔABC be a right triangle.



$$\angle B = 90^\circ$$

$$\sin A = \frac{5}{\alpha}$$

$$BC = 5k,\ AC = \alpha k$$

$$7\operatorname{cosec}A = 6\sec A$$

$$7\times\frac{AC}{BC} = 6\frac{AC}{AB}$$

$$\frac{7}{5k} = \frac{6}{AB}$$

$$AB = \frac{30k}{7}$$

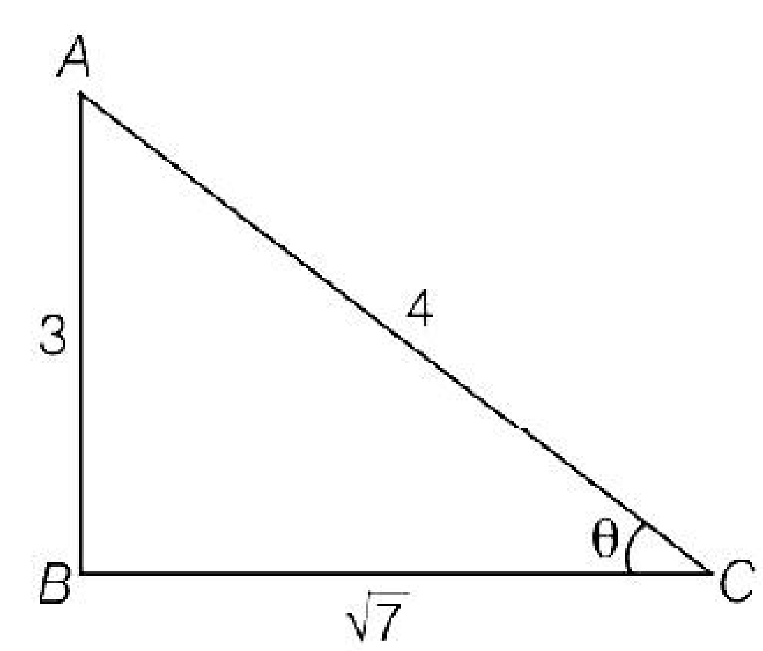
$$AB^2 + BC^2 = AC^2$$

$$\left(\frac{30}{7}k\right)^2 + (5k)^2 = \alpha^2k^2$$

$$\sqrt{\frac{900 + 49\times 25}{49}} = \alpha$$

$$\Rightarrow \quad \alpha = \sqrt{\frac{2125}{49}} = \frac{46}{7}\text{ (approx)}$$

7. Given, $4\sin\theta = 3$



$$\Rightarrow \quad \sin\theta = \frac{3}{4} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

Let $AB = 3k$ and $AC = 4k$

Apply Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$\Rightarrow \quad (3k)^2 + BC^2 = (4k)^2$

$\Rightarrow \quad BC = (\sqrt{16-9})\,k$

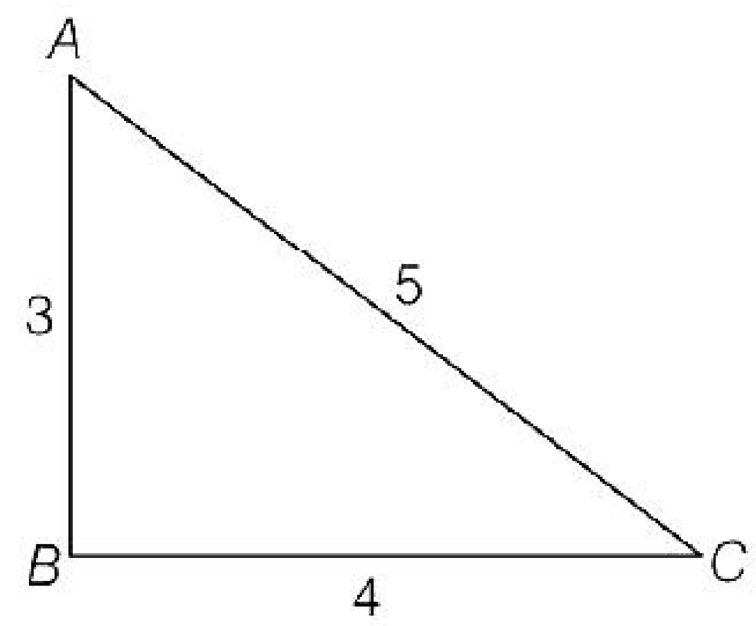
$= \sqrt{7}\,k$

$\Rightarrow \quad \cos\theta = \dfrac{\sqrt{7}\,k}{4k} = \dfrac{\sqrt{7}}{4}$

$\Rightarrow 4\sin^2\theta - 3\cos^2\theta + 2 = 4 \times \dfrac{9}{16} - 3 \times \dfrac{7}{16} + 2$

$= \dfrac{36 - 21 + 32}{16} = \dfrac{47}{16}$

8. Given, $\sin A = \dfrac{4}{5} = \dfrac{BC}{AC}$



Let $BC = 4k$, $AC = 5k$

Apply Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

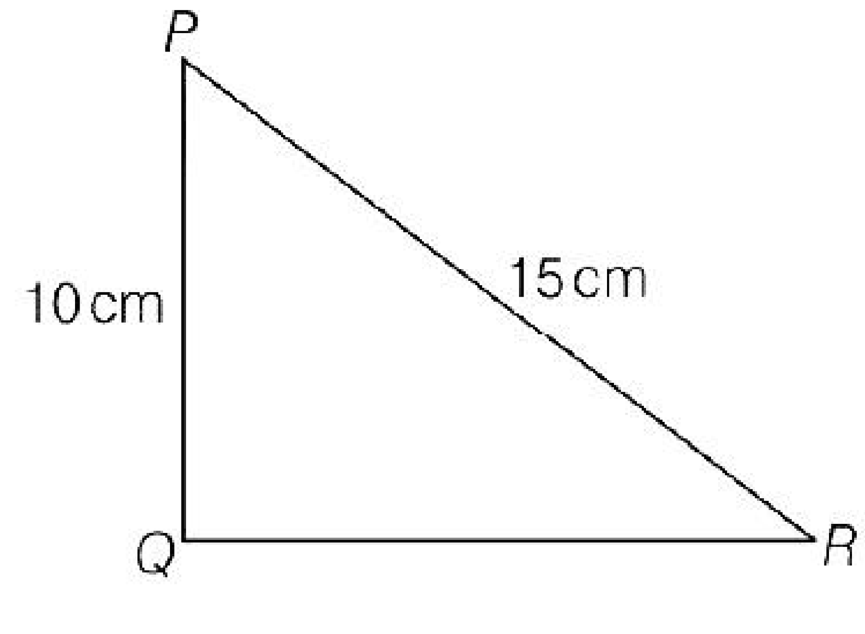
$\Rightarrow \quad AB^2 + (4k)^2 = (5k)^2$

$\Rightarrow \quad AB = \sqrt{25-16} = 3$

$\Rightarrow \quad \dfrac{1-\sin A}{1+\cos A} = \dfrac{1-\frac{4}{5}}{1+\frac{3}{5}}$

$= \dfrac{(5-4)}{5} \times \dfrac{5}{(5+3)} = \dfrac{1}{8}$

9. Given, $PQ = 10$ cm, $PR = 15$ cm



Apply Pythagoras theorem,

$PQ^2 + QR^2 = PR^2$

$\Rightarrow \quad 10^2 + QR^2 = 15^2$

$\Rightarrow \quad QR = \sqrt{225-100}$

$= \sqrt{125}$

$\therefore \quad \tan^2 p + \sec^2 p + 1$

$= \left(\dfrac{QR}{PQ}\right)^2 + \left(\dfrac{PR}{PQ}\right)^2 + 1$

$= \dfrac{125}{100} + \dfrac{225}{100} + 1$

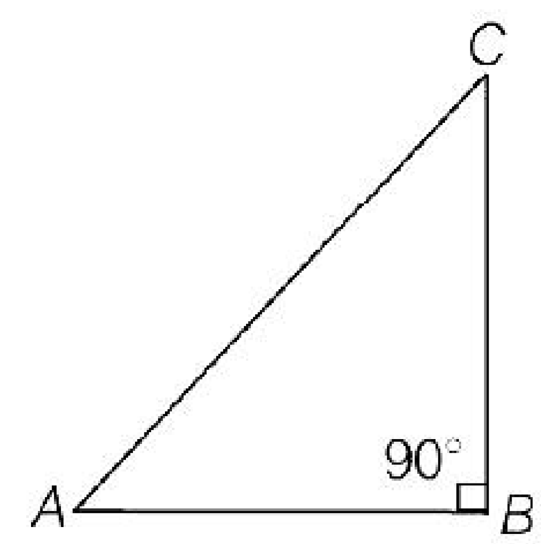
$= \dfrac{125+225+100}{100} = \dfrac{450}{100} = \dfrac{9}{2}$

10. Given, in ΔABC, $\angle B = 90^\circ$ and $AB = BC$

$\therefore \quad \angle ACB = \angle BAC$

[$\because$ angles opposite to two equal sides are equal]

In ΔABC, $\angle ABC + \angle ACB + \angle BAC = 180^\circ$



Let $\angle ACB = \angle BAC = x$

$\therefore \quad 90^\circ + x + x = 180^\circ$

[$\because$ sum of all angles of a triangle is 180°]

$\Rightarrow \quad 2x = 90^\circ \Rightarrow x = 45^\circ$

$\therefore \quad \angle ACB = \angle BAC = 45^\circ$

(i) $\sin A = \sin 45^\circ = \dfrac{1}{\sqrt{2}}$ $\left[\because \sin 45^\circ = \dfrac{1}{\sqrt{2}}\right]$

(ii) $\cos A = \cos 45^\circ = \dfrac{1}{\sqrt{2}}$ $\left[\because \cos 45^\circ = \dfrac{1}{\sqrt{2}}\right]$

11. Given, $AB = 2BC$

$\dfrac{AB}{BC} = 2$

$\tan A = \dfrac{BC}{AB} = \dfrac{1}{2}$

Let $AB = 2k$ and $BC = k$

$AC = \sqrt{4k^2 + k^2} = \sqrt{5}k$

$\sec A = \dfrac{AC}{AB} = \dfrac{\sqrt{5}k}{2k} = \dfrac{\sqrt{5}}{2}$

12. (False) Since, the value of $\cos\theta$ decrease from 1 to 0 as θ increase from 0° to 90°.

13. (True) We know that,

$\tan 30^\circ = \dfrac{1}{\sqrt{3}}$ and $\cot 30 = \sqrt{3}$

We have, $\tan^2 30^\circ + \cot^2 30^\circ$

$= \left(\dfrac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 = \dfrac{1}{3} + 3 = \dfrac{1+9}{3} = \dfrac{10}{3}$

14. Given, $\sin 2A = 2\sin A$

when, $A = 0°$, then

$$\sin(2\times 0°) = 2\sin 0°$$

$\Rightarrow$ $\sin(0°) = 2\times 0$

$\Rightarrow$ $0 = 0$

So, for $A = 0°$, given statement is true.

15. We have,

$\cot^2 30° + \text{cosec } 30° + 3\tan^2 30°$

$$= (\sqrt{3})^2 + (2) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + 2 + 3\times\frac{1}{3} = 6$$

16. Given, $\dfrac{2\tan 30°}{1-\tan^2 30°}$

$$= \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$\Rightarrow$ $\dfrac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \dfrac{2}{\sqrt{3}}\left[\dfrac{1}{3-1}\right]\times 3$

$$= \frac{1}{\sqrt{3}}\times 3 = \sqrt{3} = \tan 60°$$

17. Given, $\tan\theta + \dfrac{1}{\tan\theta} = 2$

$\Rightarrow$ $\tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow$ $\tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow$ $(\tan\theta - 1)^2 = 0$

$\Rightarrow$ $\tan\theta = 1$

$\Rightarrow$ $\tan\theta = \tan 45°$

$\Rightarrow$ $\theta = 45°$

$\Rightarrow$ $\text{cosec } 45° = \sqrt{2}$

18. Given,

$x\tan 45°\cos 60° = \sin 60°\cot 60°$

We know that,

$$\tan 45° = 1, \cos 60° = \frac{1}{2}, \sin 60° = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $\cot 60° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $x\times 1\times\dfrac{1}{2} = \dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $x = 1$

19. $7\times\dfrac{\sqrt{3}}{2} + 5\times\dfrac{1}{\sqrt{3}} + 6\times\dfrac{1}{\sqrt{3}}$

$$\left[\cos 30° = \frac{\sqrt{3}}{2}, \tan 30° = \frac{1}{\sqrt{3}}, \cot 60° = \frac{1}{\sqrt{3}}\right]$$

$$= \frac{7\times 3 + 10 + 12}{2\sqrt{3}} = \frac{43}{2\sqrt{3}}$$

20. $\tan^2\theta - 4 - 3\tan\theta$

$\tan^2\theta - 3\tan\theta - 4 = 0$

$\tan^2\theta - 4\tan\theta + \tan\theta - 4 = 0$

$\tan\theta(\tan\theta - 4) + 1(\tan\theta - 4) = 0$

$(\tan\theta + 1)(\tan\theta - 4) = 0$

$\tan\theta = -1$ [Not Applicable]

$\tan\theta = 4$ $\left[\because \sin\theta = \dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right]$

$$\sin\theta = \frac{4}{\sqrt{17}} \Rightarrow \sin^2\theta = \frac{16}{17}$$

21. Given, $\sin\alpha = \dfrac{1}{2}$ and $\cos\beta = \dfrac{1}{2}$

$\Rightarrow$ $\sin\alpha = \sin 30°$ $\left[\because \sin 30° = \dfrac{1}{2}\right]$

and $\cos\beta = \cos 60°$ $\left[\because \cos 60° = \dfrac{1}{2}\right]$

$\Rightarrow$ $\alpha = 30°$ and $\beta = 60°$

$\therefore$ $\alpha + \beta = 30° + 60° = 90°$

22. Given, $\tan A = \dfrac{1}{\sqrt{3}} = \tan 30° \Rightarrow A = 30°$

$$\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

and $\tan B = \sqrt{3} = \tan 60° \Rightarrow B = 60°$

$$[\because \tan 60° = \sqrt{3}]$$

Now, $A + B = 30° + 60° = 90°$

$\therefore$ $\tan(A+B) = \tan 90° = \infty$

23. Given, $\sqrt{3}\tan\theta = 2\sin\theta$

$$\sqrt{3}\frac{\sin\theta}{\cos\theta} = 2\sin\theta$$

$\therefore$ $\cos\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow$ $\cos\theta = \cos 30° \Rightarrow \theta = 30°$

$$= \sin^2\theta - \cos^2\theta$$

$$= \sin^2 30° - \cos^2 30°$$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1-3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

24. Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \quad \sin\theta = \cos\theta$

$\Rightarrow \quad \tan\theta = 1 = \tan 45° \Rightarrow \theta = 45°$

$\therefore \sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

25. $m \tan 30° \cot 60° = \sin 45° \cos 45°$

$$m \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\frac{m}{3} = \frac{1}{2}$$

$\therefore \quad m = \frac{3}{2}$

26. We have, $\frac{2\tan 30°}{1 + \tan^2 30°}$

$$= \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{\sqrt{3}} \times \frac{1}{2} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3 \times \sqrt{3}}{3 \times 2} = \frac{\sqrt{3}}{2} = \sin 60°$$

27. Given, $\frac{1 - \tan^2 45°}{1 + \tan^2 45°}$

$$\frac{1 - (1)^2}{1 + (1)^2} = \frac{0}{2} = 0 \quad [\because \tan 45° = 1]$$

28. Given, $\frac{\tan 30°}{\cot 60°} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times 1} = 1$

29. We know that, $A + B + C = 180°$

$\Rightarrow A = 180° - (B + C) \Rightarrow \frac{A}{2} = \frac{1}{2}[180° - (B + C)]$

$\Rightarrow \quad \frac{B + C}{2} = 90° - \frac{A}{2}$

$\Rightarrow \quad \tan\left(\frac{B + C}{2}\right) = \tan\left[90° - \frac{A}{2}\right]$

[take both sides tan]

$\Rightarrow \quad \tan\left(\frac{B + C}{2}\right) = \cot\frac{A}{2}$

30. Given, $\cos A + \cos^2 A = 1$

$\Rightarrow \quad \cos A = 1 - \cos^2 A$

$\Rightarrow \quad \cos A = \sin^2 A$

$[\because 1 - \cos^2 A = \sin^2 A] \ldots$(i)

We have to find the value of $\sin^2 A + \sin^4 A$

$\Rightarrow \quad \sin^2 A + \sin^4 A$

$\Rightarrow \quad \cos A + \cos^2 A \quad$ [from Eq (i)]

$\Rightarrow \quad 1$

31. (False) Let us take $A = 30°$ and $B = 60°$,

then LHS $= \sin(30° + 60°) = \sin 90° = 1$

RHS $= \sin 30° + \sin 60°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

LHS $\neq$ RHS

32. (False) We have,

$$\frac{1 - \sin\theta}{1 + \sin\theta} = \frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$$

[$\because$ rationalise]

$$\Rightarrow \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta} = \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$= (\sec\theta - \tan\theta)^2$$

LHS $\neq$ RHS

33. We have, $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}}$

$$= \frac{1 + \tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}}$$

$$= (1 + \tan^2 A)\frac{(\tan^2 A)}{(1 + \tan^2 A)}$$

$$= \tan^2 A$$

34. We have,

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= (1 + \tan\theta + \sec\theta)\left[1 + \frac{1}{\tan\theta} - \operatorname{cosec}\theta\right]$$

$$= (1 + \tan\theta + \sec\theta)\left[\frac{\tan\theta + 1 - \tan\theta \cdot \operatorname{cosec}\theta}{\tan\theta}\right]$$

$$= \frac{(1 + \tan\theta + \sec\theta)[\tan\theta + 1 - \sec\theta]}{\tan\theta}$$

$$\left[\because \tan\theta \cdot \operatorname{cosec}\theta = \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta} = \frac{1}{\cos\theta} = \sec\theta\right]$$

$$= \frac{(1+\tan\theta)^2 - \sec^2\theta}{\tan\theta}$$
$$= \frac{1+\tan^2\theta + 2\tan\theta - \sec^2\theta}{\tan\theta}$$
$$= \frac{1+2\tan\theta - 1}{\tan\theta}$$
$$= \frac{2\tan\theta}{\tan\theta} = 2$$

35. Given, $4x = \operatorname{cosec}\theta$ and $\frac{4}{x} = \cot\theta$

$$x = \frac{\operatorname{cosec}\theta}{4} \text{ and } \frac{1}{x} = \frac{\cot\theta}{4}$$

We have find the value of $4\left[x^2 - \frac{1}{x^2}\right]$

$$= 4\left[\left(\frac{\operatorname{cosec}\theta}{4}\right)^2 - \left(\frac{\cot\theta}{4}\right)^2\right]$$
$$= 4\left[\frac{\operatorname{cosec}^2\theta}{16} - \frac{\cot^2\theta}{16}\right]$$
$$= \frac{4}{16}[\operatorname{cosec}^2\theta - \cot^2\theta] = \frac{1}{4}$$

36. $\frac{\sin\theta\tan\theta}{1-\cos\theta} + \tan^2\theta - \sec^2\theta$

$$= \frac{\sin\theta\tan\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} - 1$$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$$= \frac{\sin\theta\cdot\tan\theta(1+\cos\theta)}{1-\cos^2\theta} - 1$$
$$= \frac{\sin^2\theta}{\cos\theta\times\sin^2\theta}(1+\cos\theta) - 1$$
$$= \frac{1}{\cos\theta} + 1 - 1$$
$$= \sec\theta$$

37. Given, $\sec\theta = x + \frac{1}{4x}$

Squaring both sides, we get

$$\sec^2\theta = \left(x + \frac{1}{4x}\right)^2$$
$$= x^2 + \frac{1}{16x^2} + \frac{1}{2}$$
$$\sec^2\theta - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$
$$\sec^2\theta - 1 = \left(x - \frac{1}{4x}\right)^2$$
$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

$[\because \sec^2\theta - 1 = \tan^2\theta]$

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

$\sec\theta + \tan\theta$

$$x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$$

38. Let $\cot\theta = x$

and $\sqrt{3}x^2 - 4x + \sqrt{3} = 0$

Here, $a = \sqrt{3}$, $b = -4$ and $c = \sqrt{3}$

$$X = \frac{-b \pm \sqrt{D}}{2a}$$

[using quadratic formula]

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{16 - 4\cdot\sqrt{3}\cdot\sqrt{3}}}{2\times\sqrt{3}} = \frac{4 \pm \sqrt{4}}{2\sqrt{3}}$$
$$\Rightarrow x = \frac{4+2}{2\sqrt{3}}, \frac{4-2}{2\sqrt{3}}$$
$$\Rightarrow \cot\theta = \frac{6}{2\sqrt{3}}, \frac{2}{2\sqrt{3}} \Leftrightarrow \cot\theta = \sqrt{3}, \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}, \sqrt{3}$$
$$\Rightarrow \cot^2\theta + \tan^2\theta = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$
$$= 3 + \frac{1}{3} = \frac{10}{3}$$

39. Given, $\tan\theta + \sec\theta = I$ ...(i)

$$\Rightarrow \frac{(\tan\theta + \sec\theta)(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)} = I$$
$$\Rightarrow \frac{(\sec^2\theta - \tan^2\theta)}{(\sec\theta - \tan\theta)} = I$$
$$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = I$$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$$\Rightarrow \sec\theta - \tan\theta = \frac{1}{I} \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2\sec\theta = I + \frac{1}{I}$$
$$\Rightarrow \sec\theta = \frac{I^2+1}{2I}$$

40. Given, $\tan A = a \tan B$ and $\sin A = b \sin B$

$$\cot B = \frac{a}{\tan A} \text{ and } \operatorname{cosec} B = \frac{b}{\sin A}$$

We know that,

$$\operatorname{cosec}^2 B - \cot^2 B = 1$$

$$\left(\frac{b}{\sin A}\right)^2 - \left(\frac{a}{\tan A}\right)^2 = 1$$

$$\frac{b^2}{\sin^2 A} - \frac{a^2}{\tan^2 A} = 1$$

$$\frac{1}{\sin^2 A}[b^2 - a^2 \cos^2 A] = 1$$

$$b^2 - a^2 \cos^2 A = \sin^2 A$$

$$b^2 = 1 - \cos^2 A + a^2 \cos^2 A$$

$$\cos^2 A(a^2 - 1) = (b^2 - 1)$$

$$\Leftrightarrow \quad \cos^2 A = \frac{b^2 - 1}{a^2 - 1}$$

41. Given, $\operatorname{cosec} A - \cot A = q$

$$= (\operatorname{cosec} A - \cot A)^2 = q^2$$

$$= \operatorname{cosec}^2 A + \cot^2 A - 2\operatorname{cosec} A \cdot \cot A = q^2$$

$\therefore$ Put the value of q^2 in $\frac{q^2 - 1}{q^2 + 1} + \cos A$.

$$= \frac{\operatorname{cosec}^2 A + \cot^2 A - 2\operatorname{cosec} A \cdot \cot A - 1}{\operatorname{cosec}^2 A + \cot^2 A - 2\operatorname{cosec} A \cdot \cot A + 1} + \cos A$$

$$= \frac{2\cot^2 A - 2\operatorname{cosec} A \cdot \cot A}{2\operatorname{cosec}^2 A - 2\operatorname{cosec} A \cdot \cot A} + \cos A$$

$$= \frac{2\cot A}{2\operatorname{cosec} A}\left[\frac{\cot A - \operatorname{cosec} A}{\operatorname{cosec} A - \cot A}\right] + \cos A$$

$$= \cos A(-1) + \cos A = 0$$

42. Given, $x = r \sin A \cos B$ ...(i)

$$y = r \sin A \sin B$$

$$\Leftrightarrow \quad y^2 = r^2 \sin^2 A \cdot \sin^2 B \quad \text{...(ii)}$$

$$z = r \cos A$$

$$\Leftrightarrow \quad z^2 = r^2 \cos^2 A \quad \text{...(iii)}$$

On adding Eqs. (i), (ii) and (iii), we get

$$x^2 + y^2 + z^2$$
$$= r^2 \sin^2 A \cos^2 B + r^2 \sin^2 A \cdot \sin^2 B + r^2 \cos^2 A$$

$$x^2 + y^2 + z^2 = r^2 \sin^2 A(\cos^2 B + \sin^2 B) + r^2 \cos^2 A$$

$$x^2 + y^2 + z^2 = r^2 \sin^2 A + r^2 \cos^2 A$$
$$= r^2 [\sin^2 A + \cos^2 A]$$
$$= r^2$$

43. We know that,

$$\sin^2 A + \cos^2 A = 1$$
$$1 + \cos^2 A = 1$$
$$\cos^2 A = 0$$
$$\sin^2 A - \cos^2 A = 1 - 0 = 1$$

44. P $\rightarrow$ 4; Q $\rightarrow$ 1; R $\rightarrow$ 2; S $\rightarrow$ 3

(P) $\frac{\sin 0^\circ}{\cos 90^\circ} + \sin 45^\circ = 0 + \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

(Q) $\cos 60^\circ - \sin 60^\circ = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1 - \sqrt{3}}{2}$

(R) $\sec 30^\circ \sin 60^\circ + \cos 45^\circ \operatorname{cosec} 60^\circ$

$$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}} = 1 + \frac{\sqrt{2}}{\sqrt{3}}$$

(S) $\frac{\cos^3 30^\circ - \cos^3 60^\circ}{\sin^3 60^\circ - \sin^3 30^\circ} = \frac{\left(\frac{\sqrt{3}}{2}\right)^3 - \left(\frac{1}{2}\right)^3}{\left(\frac{\sqrt{3}}{2}\right)^3 - \left(\frac{1}{2}\right)^3} = 1$

45. P $\rightarrow$ 4; Q $\rightarrow$ 1; R $\rightarrow$ 2; S $\rightarrow$ 3

(P) $1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta}$

$$= 1 + \frac{\operatorname{cosec}^2 \theta - 1}{1 + \operatorname{cosec} \theta}$$

$$= 1 + \frac{(\operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 1)}{(\operatorname{cosec} \theta + 1)}$$

$$= 1 + \operatorname{cosec} \theta - 1$$

$$= \operatorname{cosec} \theta = \sec \theta \cot \theta$$

(Q) $\frac{\cos \theta}{\operatorname{cosec} \theta + 1} + \frac{\cos \theta}{\operatorname{cosec} \theta - 1}$

$$= \cos \theta \left(\frac{1}{\operatorname{cosec} \theta + 1} + \frac{1}{\operatorname{cosec} \theta - 1}\right)$$

$$= \cos \theta \left(\frac{\operatorname{cosec} \theta - 1 + \operatorname{cosec} \theta + 1}{\operatorname{cosec}^2 \theta - 1}\right)$$

$$= \frac{\cos \theta (2 \operatorname{cosec} \theta)}{\cot^2 \theta}$$

$$= 2 \frac{\cot \theta}{\cot^2 \theta}$$

$$= 2 \tan \theta$$

(R) $\tan^2\theta + \cot^2\theta - 2 = (\tan\theta - \cot\theta)^2$

$$= \left(\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{\sin^2\theta - \cos^2\theta}{\cos\theta\,\sin\theta}\right)^2$$

(S) $\dfrac{1-\cos\theta}{1+\cos\theta} = \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= (\operatorname{cosec}\theta - \cot\theta)^2$$

46. Assertion

We have,

$$2\cos\theta = a + \frac{1}{a}$$

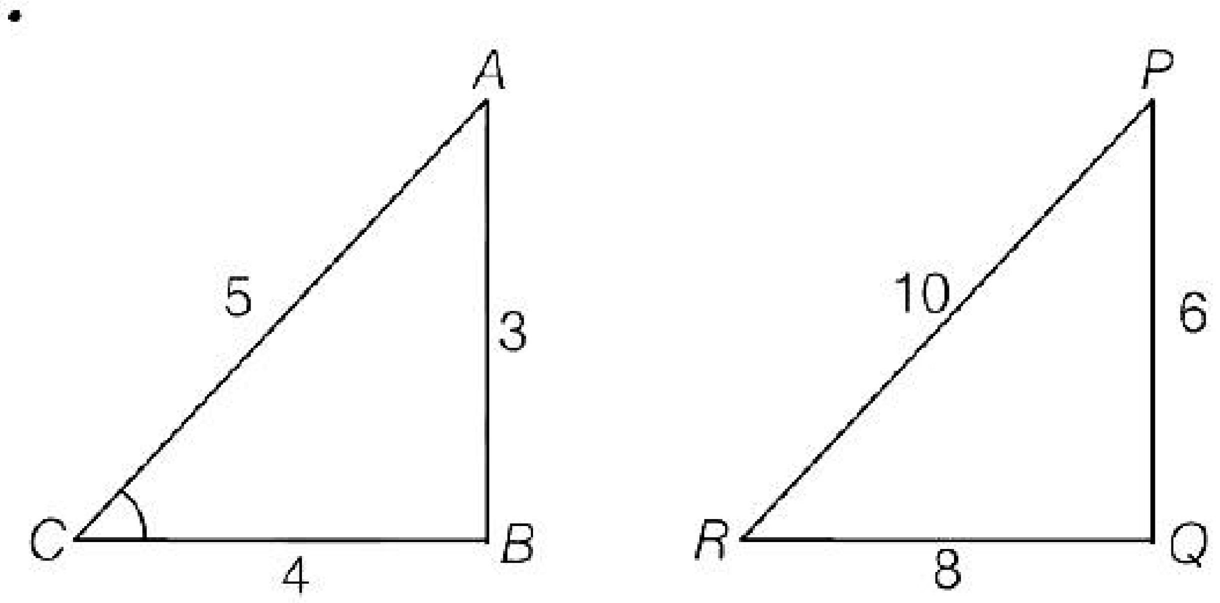
It is not possible because $a > 0$.

$\therefore \quad \cos\theta \le 1$

and Reason is true.

Assertion is false but reason is true.

47.



In ΔABC and in ΔPQR

$$\sin C = \frac{AB}{AC}$$

and $\quad \sin R = \dfrac{6}{10}$

$\Rightarrow \quad \sin C = \dfrac{3}{5}$

$\Rightarrow \quad \sin R = \dfrac{3}{5}$

$\therefore$ Trigonometric ratio does not depend on the size of the triangle.

Assertion is True and Reason is True but Reason is not the correct explanation of Assertion.

48. We know that,

$$\cos^2\theta \le 1$$

$$\sec^2\theta = \frac{4xy}{(x+y)^2} \ge 1$$

$\Rightarrow \quad 4xy \ge (x+y)^2$

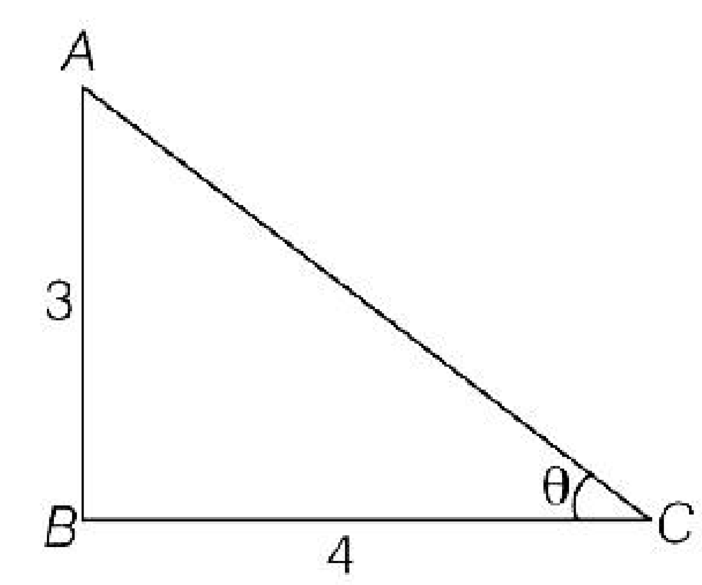
$\Rightarrow \quad (x-y)^2 \le 0$

Assertion is true and Reason is true and is the correct explanation of assertion.

49. Assertion

Given, $\quad \tan\theta = \dfrac{3}{4}$

$$\tan\theta = \frac{p}{b}$$



Perpendicular = 3 units base = 4 units

Apply Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad AC^2 = 3^2 + 4^2$

$\Rightarrow \quad AC = \sqrt{9+16} = 5$ units

Reason is true and correct explanation of the assertion.

50. Assertion

$\cos 30^\circ = \dfrac{\sqrt{3}}{2}$ and $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$

Assertion is true

Reason $\sin 2\theta = \sin\theta + \sin\theta$

This statement is false for an acute angle.

$\theta = 30^\circ \Leftrightarrow \sin 2\times 30^\circ = \sin 30^\circ + \sin 30^\circ$

$$\frac{\sqrt{3}}{2} = \frac{1}{2} + \frac{1}{2} \qquad \text{[False]}$$

Assertion is true but Reason is false.

51. **Assertion** $\cos 60° - \sin 60°$

$$\frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2} \quad \text{[It is negative]}$$

Assertion is true and Reason is false.

52. **Assertion** $\dfrac{\sin^2\theta - \cos^2\theta}{\cos^2\theta - \sin^2\theta} = -1$

Assertion is true and Reason is false.

53. We have, $\cos^2 A - \sin^2 A = 1$

Put $A = 45°$, we get

$$\cos^2 45° - \sin^2 45° = 0$$

and $\tan^2 A - \sec^2 A = 1$

Put $A = 45°$, $\tan^2 45° - \sec^2 45° = -1$

These are not trigonometric but identities.

Assertion is false but Reason is true.

54. **Assertion**

$$(\cot\theta + 3)(3\cot\theta + 1)$$
$$= 3\cot^2\theta + 3 + 9\cot\theta + \cot\theta$$
$$= 3(1 + \cot^2\theta) + \cot\theta(9 + 1)$$
$$= 3\operatorname{cosec}^2\theta + 10\cot\theta$$

Reason True

Hence, Assertion and Reason are true and correct explanation for Assertion.

55. We have,

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$
$$\Rightarrow \quad x(\sec\theta - \tan\theta) = 1$$
$$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{x}$$

Thus, we have

$$\sec\theta + \tan\theta = x$$

and $$\sec\theta - \tan\theta = \frac{1}{x}$$

Adding and subtracting these two equations, we get

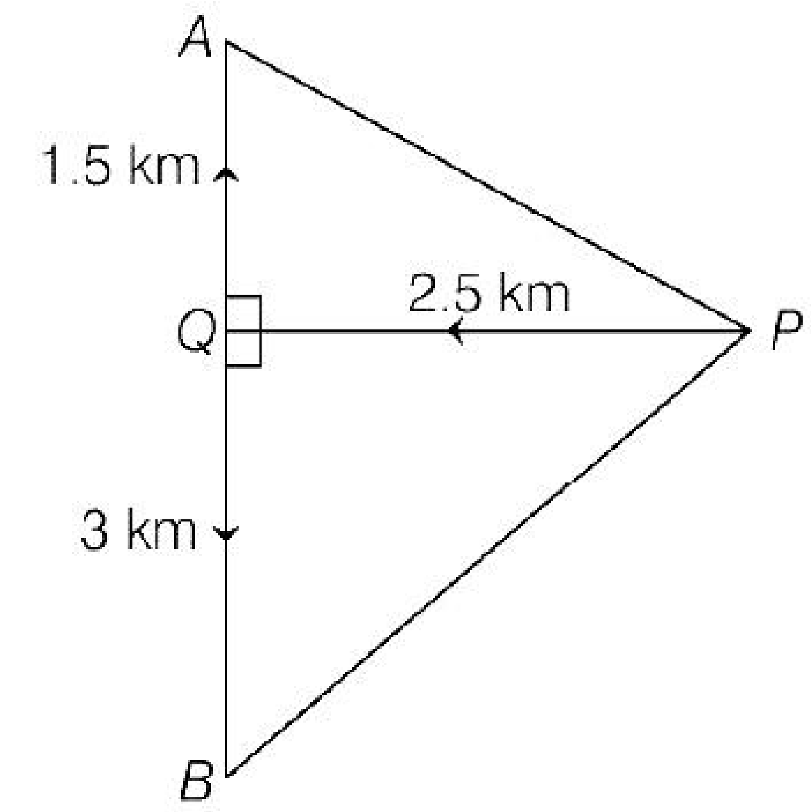
$$2\sec\theta = x + \frac{1}{x} \text{ and } 2\tan\theta = x - \frac{1}{x}$$
$$\Rightarrow \sec\theta = \frac{1}{2}\left(x + \frac{1}{x}\right) \text{ and } \tan\theta = \frac{1}{2}\left(x - \frac{1}{x}\right)$$

Now, $\sin\theta = \dfrac{\tan\theta}{\sec\theta}$

$$\Rightarrow \quad \sin\theta = \frac{\frac{1}{2}\left(x - \frac{1}{x}\right)}{\frac{1}{2}\left(x + \frac{1}{x}\right)} = \frac{x^2 - 1}{x^2 + 1}$$

Assertion is true but Reason is false.

56. (i) In ΔAPQ, $\angle APQ = \theta$



$$\tan\theta = \frac{AQ}{QP} \Rightarrow \tan\theta = \frac{1.5}{2.5} = \frac{3}{5}$$

(ii) $\cot B = \dfrac{BQ}{PQ}$

$$\cot B = \frac{3}{25} \times 10 = \frac{3 \times 2}{5} = \frac{6}{5}$$

(iii) $\tan A = \dfrac{PQ}{AQ} = \dfrac{2.5}{1.5} = \dfrac{5}{3}$

(iv) $\sec A = \dfrac{AP}{AQ} = \dfrac{\sqrt{AQ^2 + PQ^2}}{AQ}$

$$= \frac{\sqrt{(1.5)^2 + (2.5)^2}}{1.5}$$

$$\sec A = \frac{\sqrt{2.25 + 6.25}}{1.5} = \frac{\sqrt{8.5}}{1.5} = \frac{3}{1.5} \text{ (approx)}$$

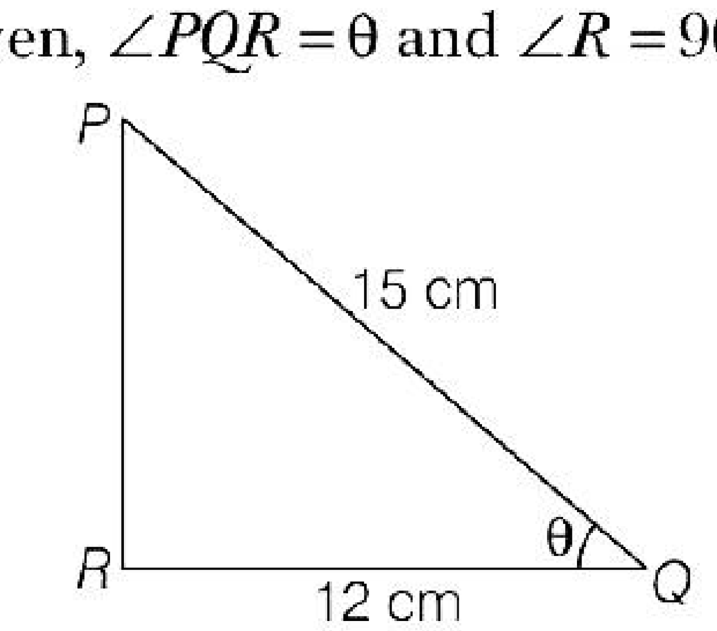
$$\sec A = \frac{3}{15} \times 10 = 2$$

(v) $\operatorname{cosec} B = \dfrac{BP}{PQ} = \dfrac{\sqrt{(3)^2 + (2.5)^2}}{2.5}$

$$\frac{\sqrt{9 + 6.25}}{25} = \frac{4}{25} \times 10 \text{ (approx)}$$

$$\operatorname{cosec} B = \frac{8}{5}$$

57. (i) Given, $\angle PQR = \theta$ and $\angle R = 90°$



$$\cos\theta = \frac{QR}{PQ} = \frac{12}{15} = \frac{4}{5}$$

(ii) $\sec\theta = \frac{PQ}{RQ} = \frac{15}{12} = \frac{5}{4}$

(iii) $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\tan\theta}{\sec^2\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \cos^2\theta$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$[\because 1 + \tan^2\theta = \sec^2\theta]$

$= \sin\theta \times \cos\theta$

$= \frac{PR}{PQ} \times \frac{RQ}{PQ} = \frac{9}{15} \times \frac{12}{15}$

$[PR = \sqrt{225-144} = \sqrt{81} = 9]$

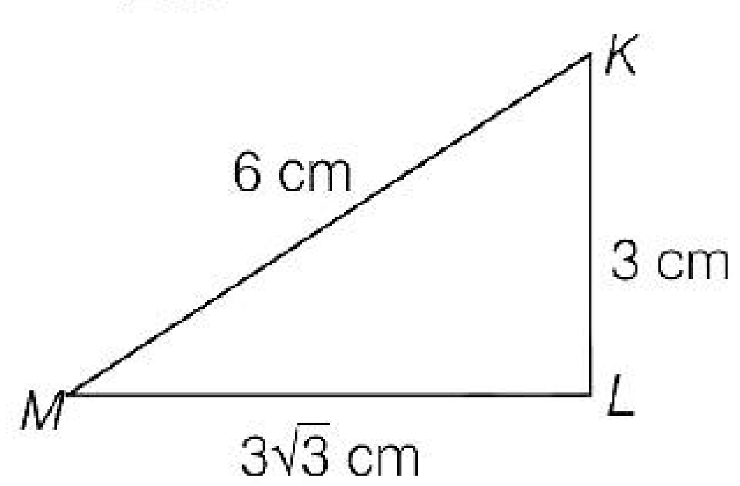
$= \frac{12}{25}$

(iv) $\cot^2\theta - \text{cosec}^2\theta = -1$ $[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

(v) $\sin^2\theta + \cos^2\theta = \left(\frac{PR}{PQ}\right)^2 + \left(\frac{QR}{PQ}\right)^2$

$= \left(\frac{9}{15}\right)^2 + \left(\frac{18}{15}\right)^2 = \frac{81+144}{225} = \frac{225}{225} = 1$

58. (i) $\tan M = \frac{KL}{LM}$



$\tan M = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan M = \tan 30°$

$\angle M = 30°$

(ii) $\tan K = \frac{3\sqrt{3}}{3}$

$\tan K = \sqrt{3}$

$\tan K = \tan 60°$

$\therefore \angle K = 60°$

(iii) $\tan M = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

(iv) $\sec^2 M - 1 = \left(\frac{KM}{LM}\right)^2 - 1$

$\sec^2 M - 1 = \frac{36}{27} - 1 = \frac{36-27}{27} = \frac{9}{27}$

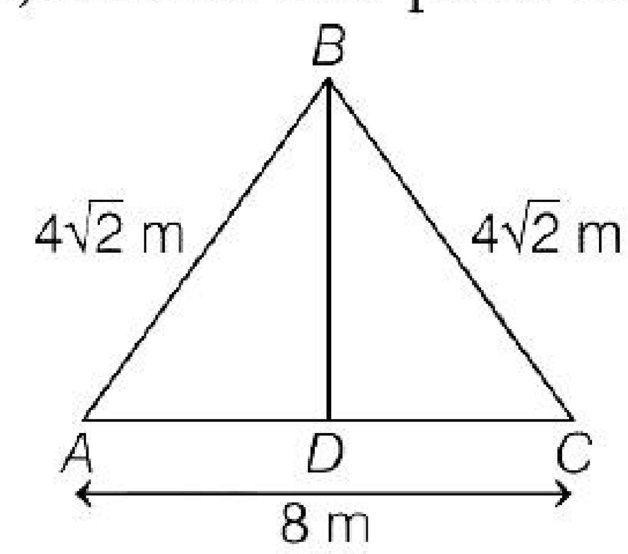
$\sec^2 M - 1 = \frac{1}{3}$

$\tan M = \frac{1}{\sqrt{3}}$

$\tan^2 M = \frac{1}{3} \Rightarrow \sec^2 M - 1 = \tan^2 M$

(v) $\frac{\tan^2 45° - 1}{\tan^2 45° + 1} = \frac{1-1}{1+1} = 0$

59. (i) Given, D is the mid-point of AC.



$AD = DC = \frac{AC}{2} = \frac{8}{2} = 4$ m

$BD = \sqrt{(BC)^2 - (DC)^2}$

[Apply Pythagoras theorem in ΔACD]

$BD = \sqrt{(4\sqrt{2})^2 - (4)^2}$

$= \sqrt{32-16}$

$= \sqrt{16}$ m $= 4$ m

(ii) In ΔABD, $\angle D = 90°$

$\tan A = \frac{BD}{AD}$

$\tan A = \frac{4}{4} = 1$

$\angle A = 45°$ $[\because \tan 45° = 1]$

(iii) $\tan C = \frac{BD}{DC}$

$\tan C = \frac{4}{4} = 1$

$\angle C = 45°$

(iv) $\sin A + \cos C = \frac{BD}{BA} + \frac{CD}{BC}$

$= \frac{4}{4\sqrt{2}} + \frac{4}{4\sqrt{2}}$

$= \frac{8}{4\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(v) $\tan^2 C + \tan^2 A$

$= \left(\frac{BD}{CD}\right)^2 + \left(\frac{BD}{AD}\right)^2$

$= \left(\frac{4}{4}\right)^2 + \left(\frac{4}{4}\right)^2 = 1 + 1 = 2$